# Exercises: Micro-scale flows

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Below are four exercises that go back on what was seen in the lecture. The calculations in each of them are simple. Try to do them with minimal math. Give the answers to one significant figure.

## 1 Which flow is a low Re flow?

Observe the two movies marked Taylor 1 and Taylor 2.

- Which one of the situations corresponds to a low Re flow? How do you know it? There are several indices.
- What is the main fluid property that varies between the two movies?
- In the high Reynolds number case: Estimate the viscous diffusion time from the formula, supposing that the liquid is water. How does it compare with the time that you would extract from the movie? Why?

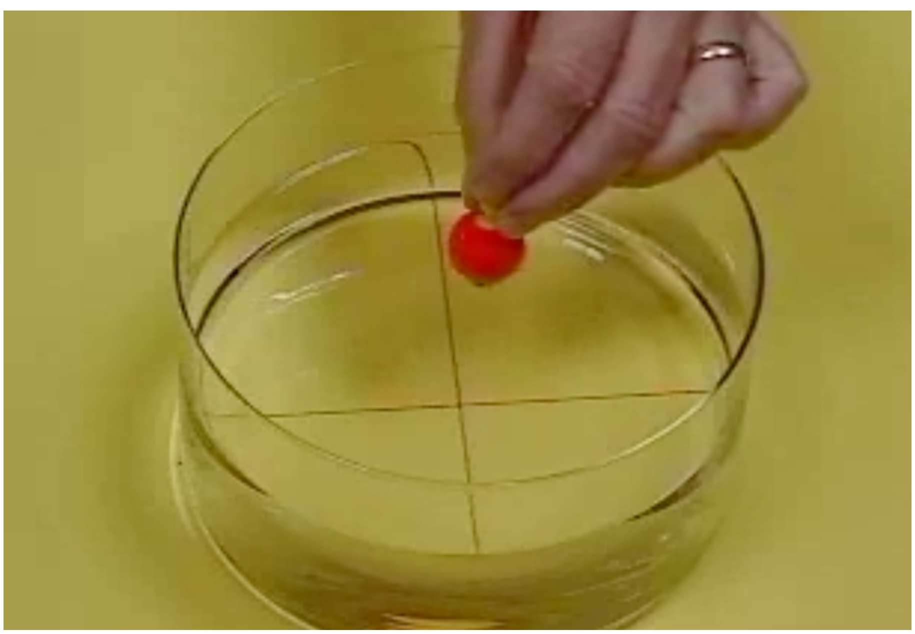
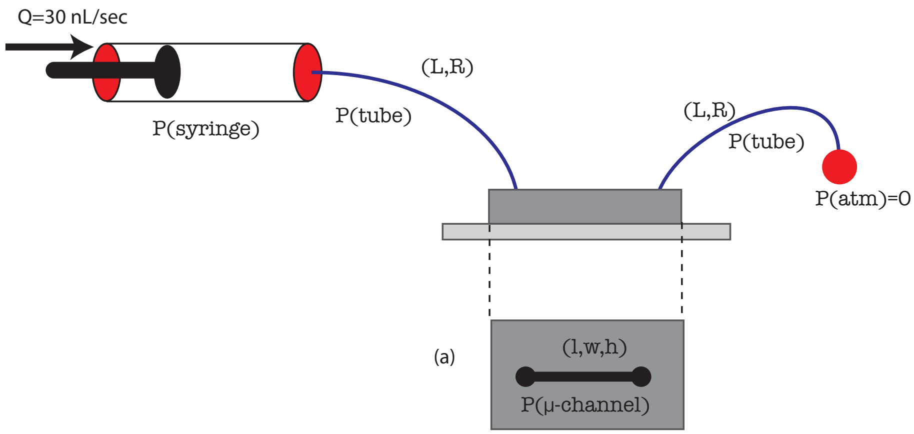


**Fig. 1**- Screen shot from movies.

## 2 Hydrodynamic resistance



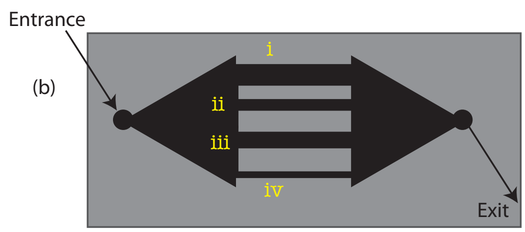




**Fig. 2**- Two microfluidic circuits.

1. Consider the microfluidic circuit in Fig. 1(a). Suppose that the liquid that is pushed is water ($\mu = 10^{-3}$ Pa.s). What is the highest Reynolds number in this circuit? Can we talk about Stokes flows everywhere in the system? The dimensions of each element are the following:

| Element | Width or Radius | Length |
|---|---|---|
| Syringe | 2 mm | 4 cm |
| Tube ($\times 2$) | 0.5 mm | 20 cm |
| Microchannel | width=100 $\mu$m | 2 cm |
| | height=20 $\mu$m | |

2. Calculate the pressure drop in each one of the elements of the circuit. Which element provides the dominant resistance? What would the flow rate be if the syringe pump is replaced with a pressure source that applies $P = 10^4$ Pa?

3. Now replace the single microfluidic channel by the network shown in part (b). The heights of all channels are all the same ($h = 20$ $\mu$m). The widths are: (i) 400 $\mu$m. (ii) 200 $\mu$m. (iii) 300 $\mu$m (iv) 100 $\mu$m. What is the flow in each channel? What is the effect of the location of each channel? What would the total flow rate be if the syringe pump is replaced with a pressure source with $P = 10^4$ Pa?.

## 3 The effect of the presence of a bubble

Consider a microchannel such as the one shown in Fig. 2a. Imagine that an air bubble is stuck in the syringe. Its volume is 1 $\mu$L (ignore surface tension effects).

1. What is the radius that corresponds to this bubble volume?
2. By how much does the bubble compress for a flow $Q = 30$ nL/sec? How does that change if $Q = 300$ nL/sec? (Recall perfect gas law).
3. Now suppose that the flow is suddenly switched off and the bubble wants to return to the volume it has at atmospheric pressure. From the resistance of the channels calculated above and supposing that the pressure in the bubble is constant during this time (for example take the average value of pressure), estimate how long it takes for the flow to stop.
4. What would the "exponential" time be if we calculate it from the RC equivalent circuit?

# 4 Mixing

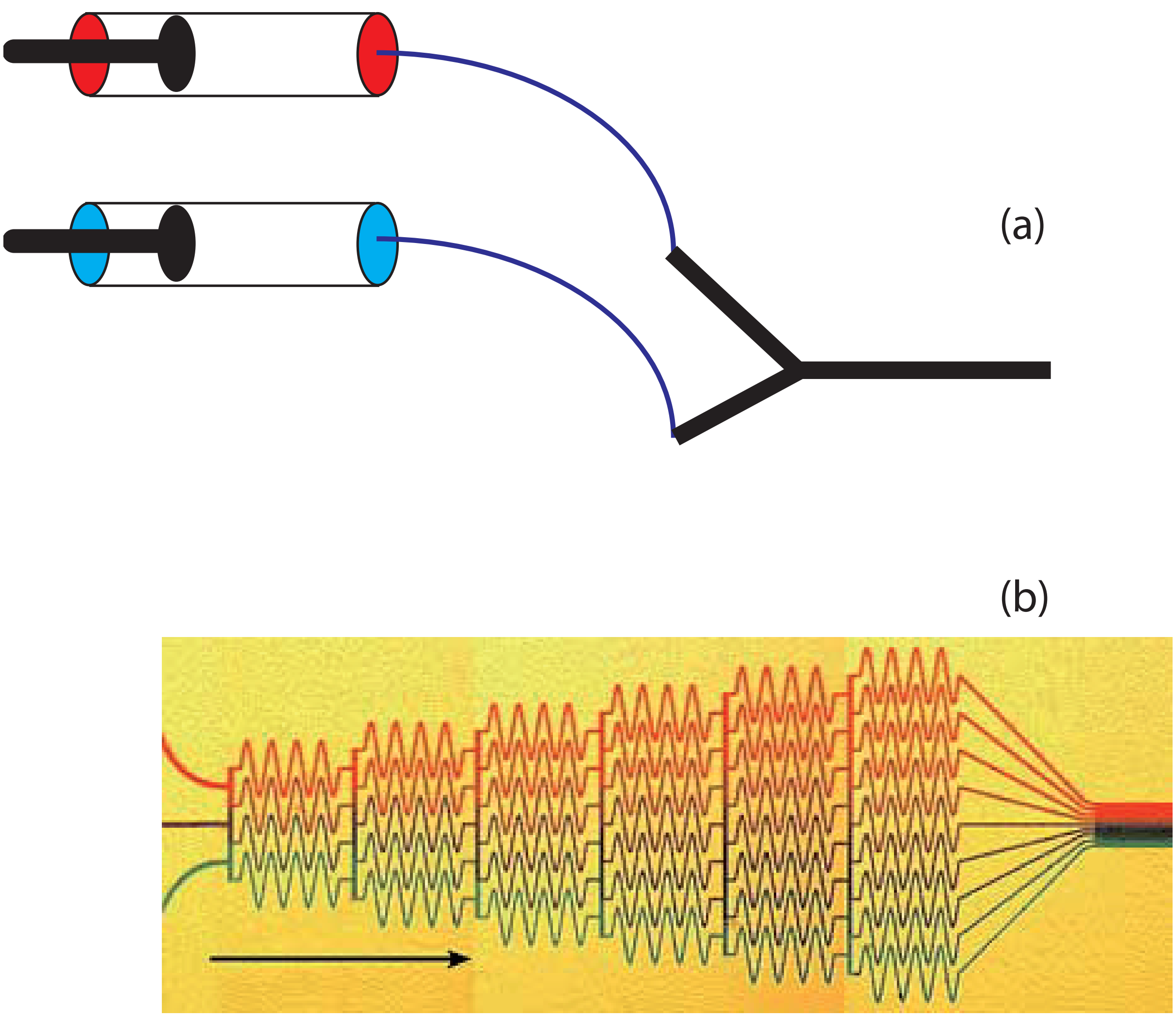



**Fig. 3**- Mixing in microchannels.

1. Consider the channel in Fig. 3a. The main channel has the same dimensions as the channel in Fig. 2a but it has two inputs. Each of the inputs is connected to a syringe pushing at $Q_1 = Q_2 = 15$ nL/sec, with one pushing blue dye and the other pushing red dye. Considering that the diffusion coefficients of the red and blue dyes are $D = 10^{-9}$ m$^2$/s, what is the percentage of the two two dyes that mixes in a channel that is 2 cm long?

2. The channel that is shown in part (b) was designed to generate a gradient of dye in the downstream channel (see the wide channel on the right hand side). How does it work?