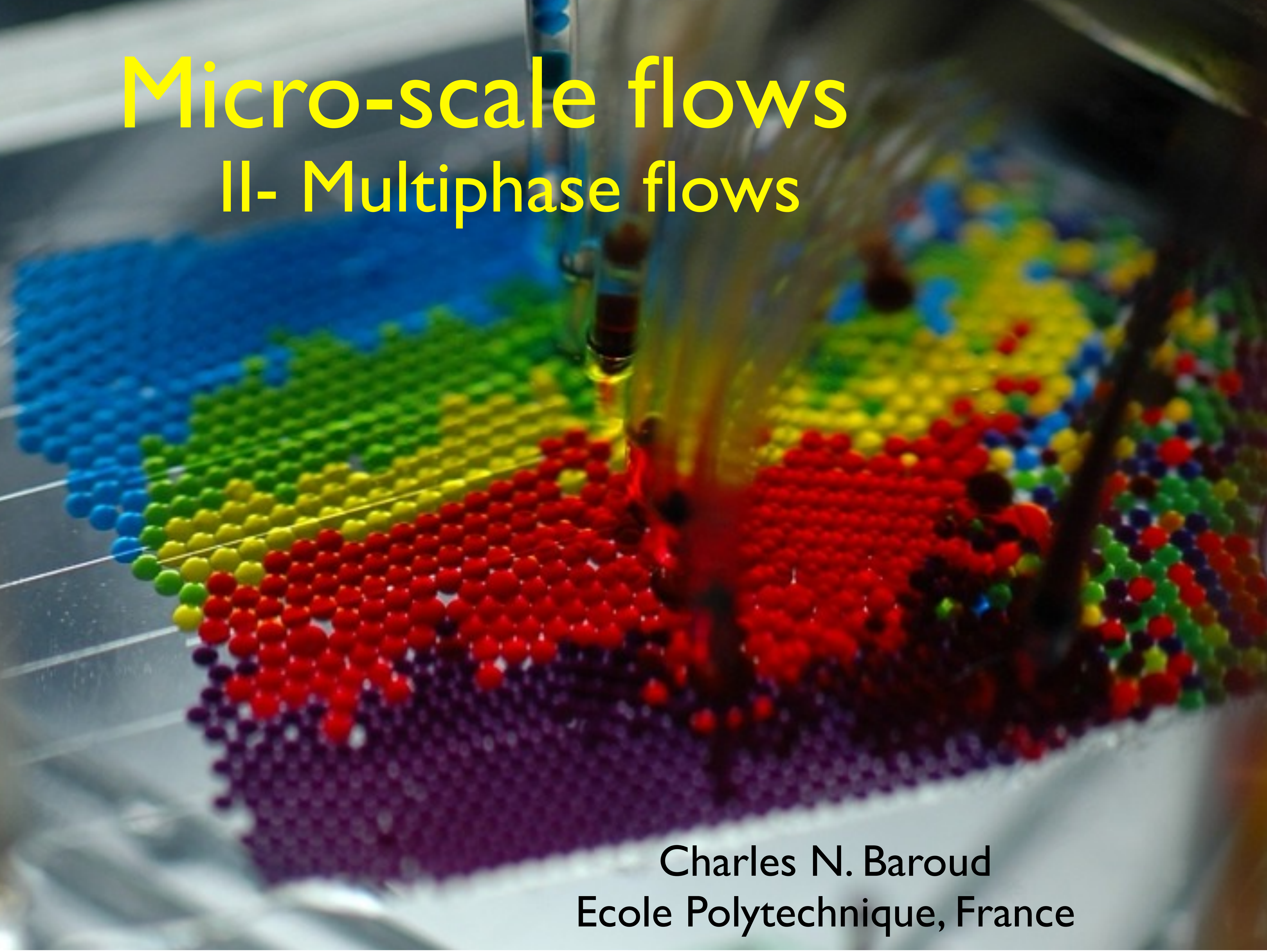
# Micro-scale flows

## II- Multiphase flows



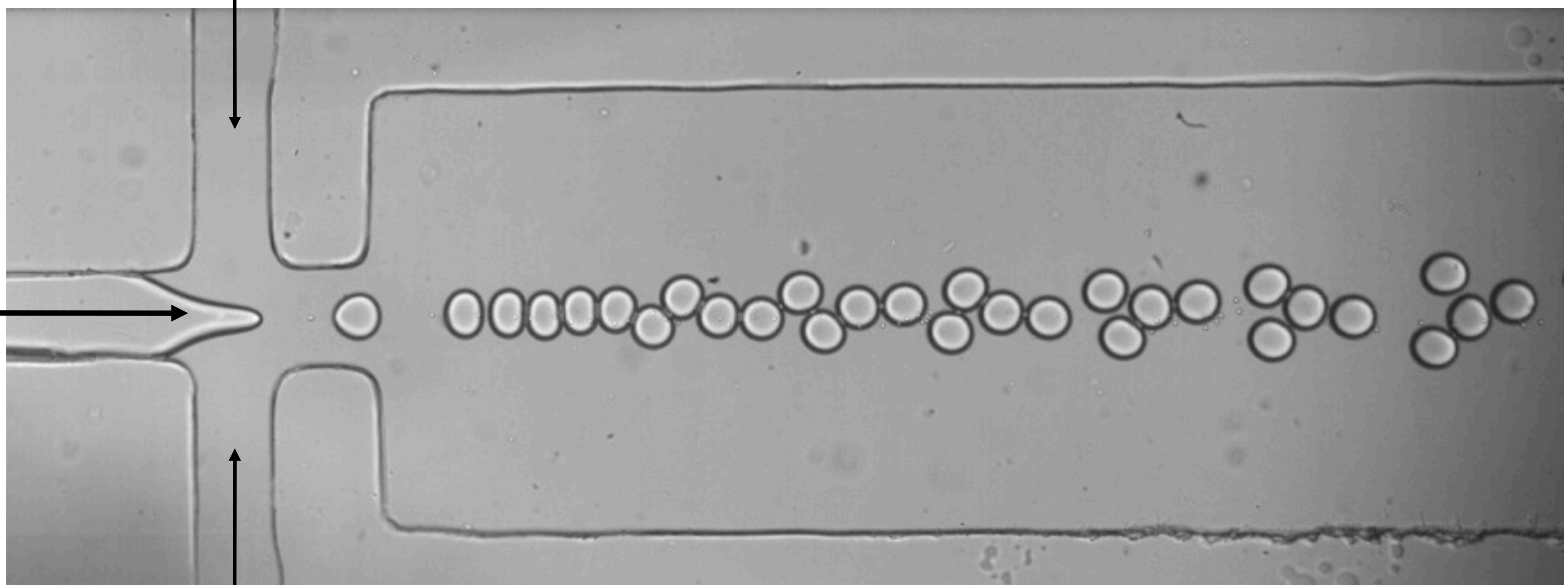
Charles N. Baroud
Ecole Polytechnique, France

# Multiphase flows

The bad news:

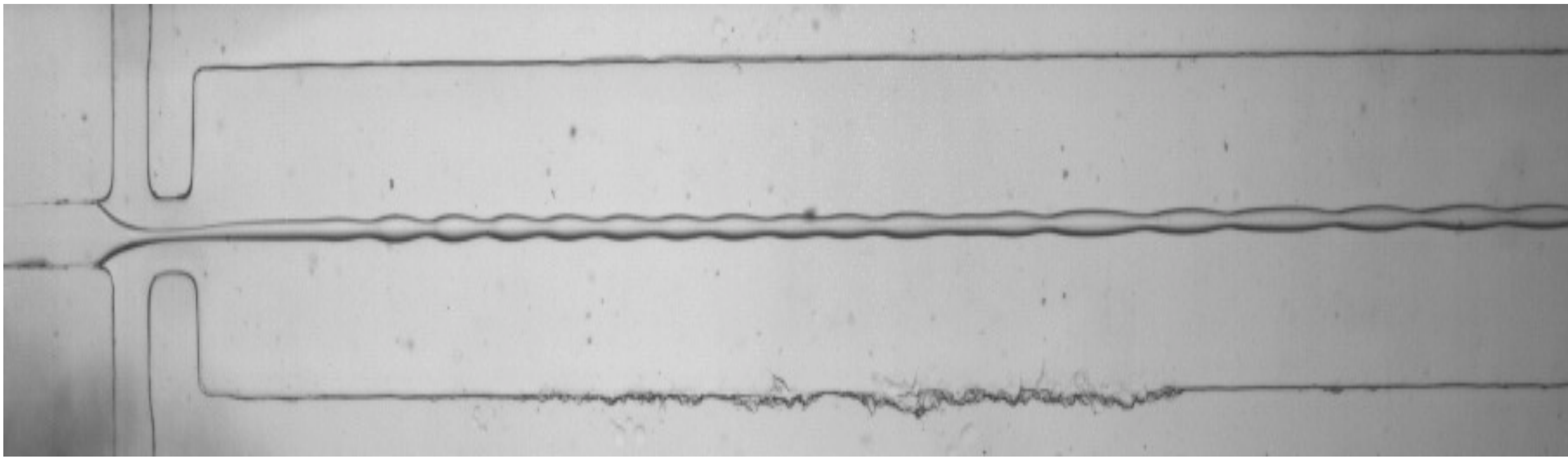
Presence of drops and bubbles introduces nonlinearities into Stokes equation

# Sign of nonlinearity




Cordero & Baroud, unpublished

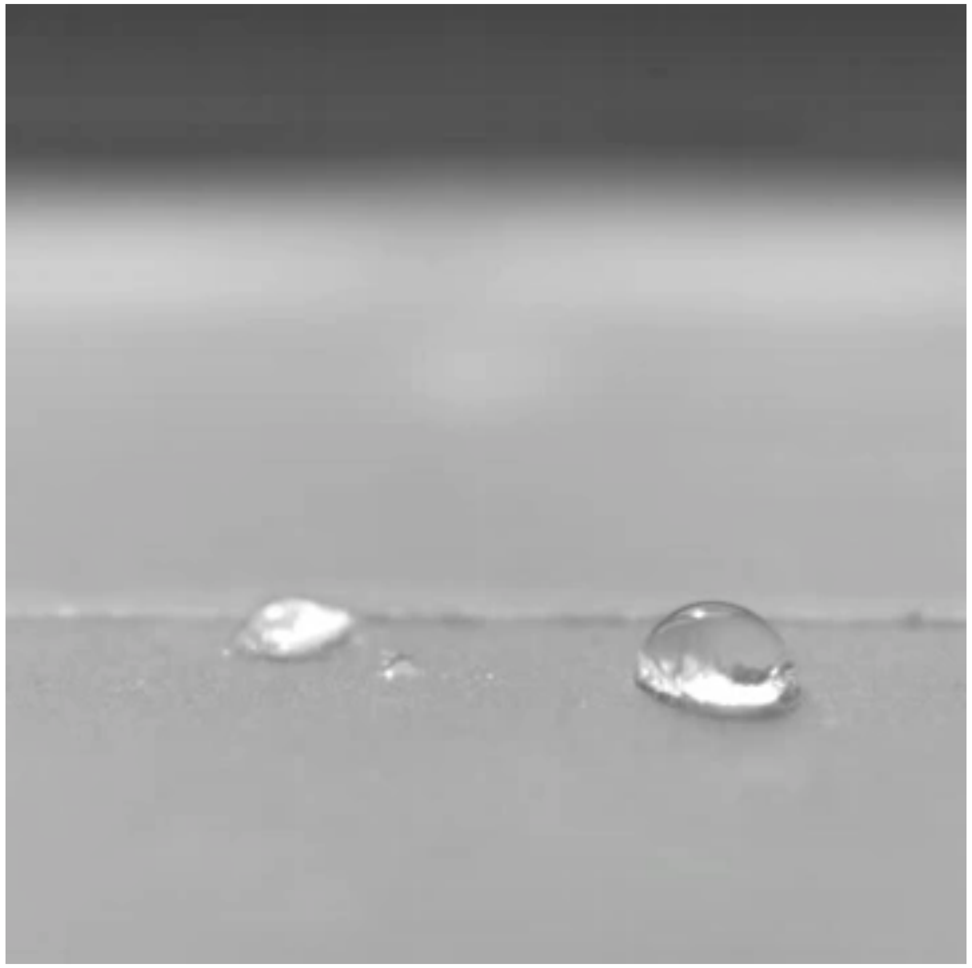
# Slightly different conditions



Cordero & Baroud, unpublished

# Surface tension

- Two ways to think about surface tension:



- Force per unit length

$$\gamma = [\mathrm{N/m}]$$

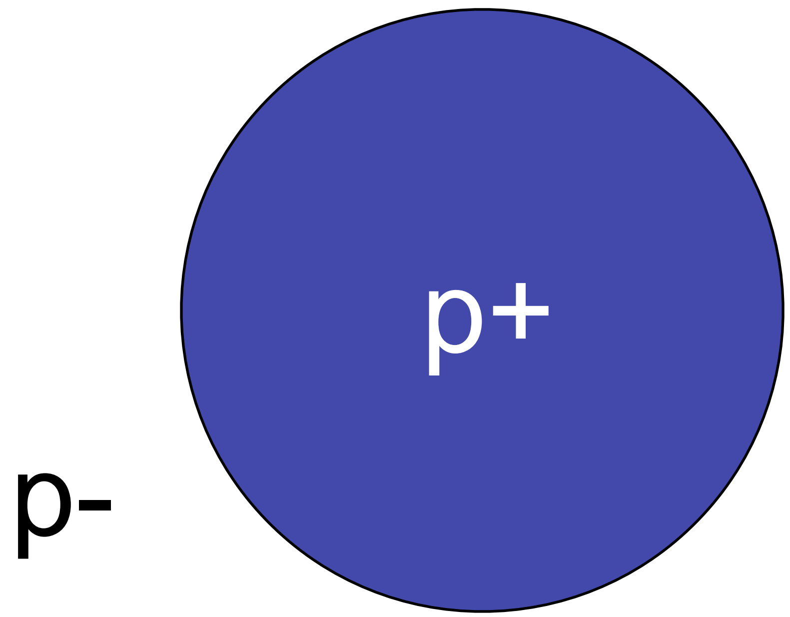
- Energy per unit area

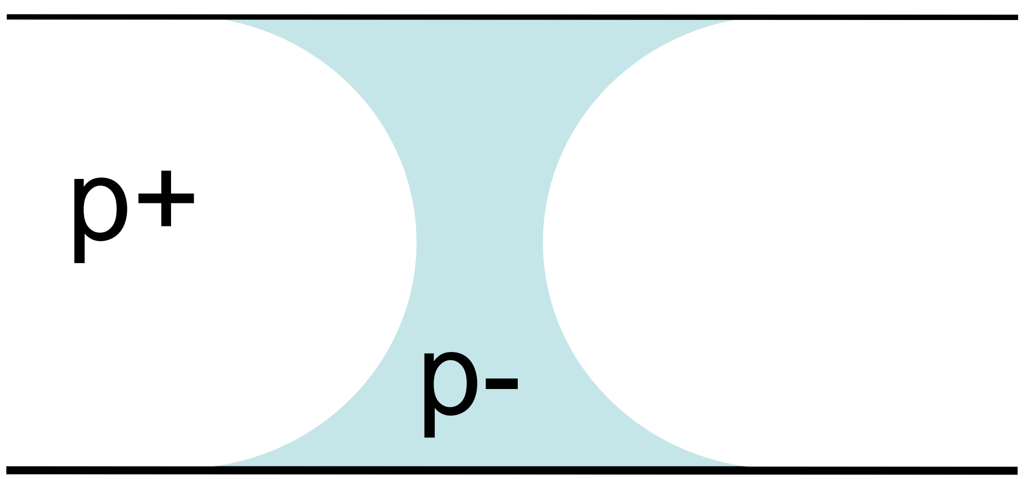
$$\gamma = [\mathrm{J/m}]^2$$

Gives cohesion to liquids

# Consequence

$$\Delta P_{cap} = \gamma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$



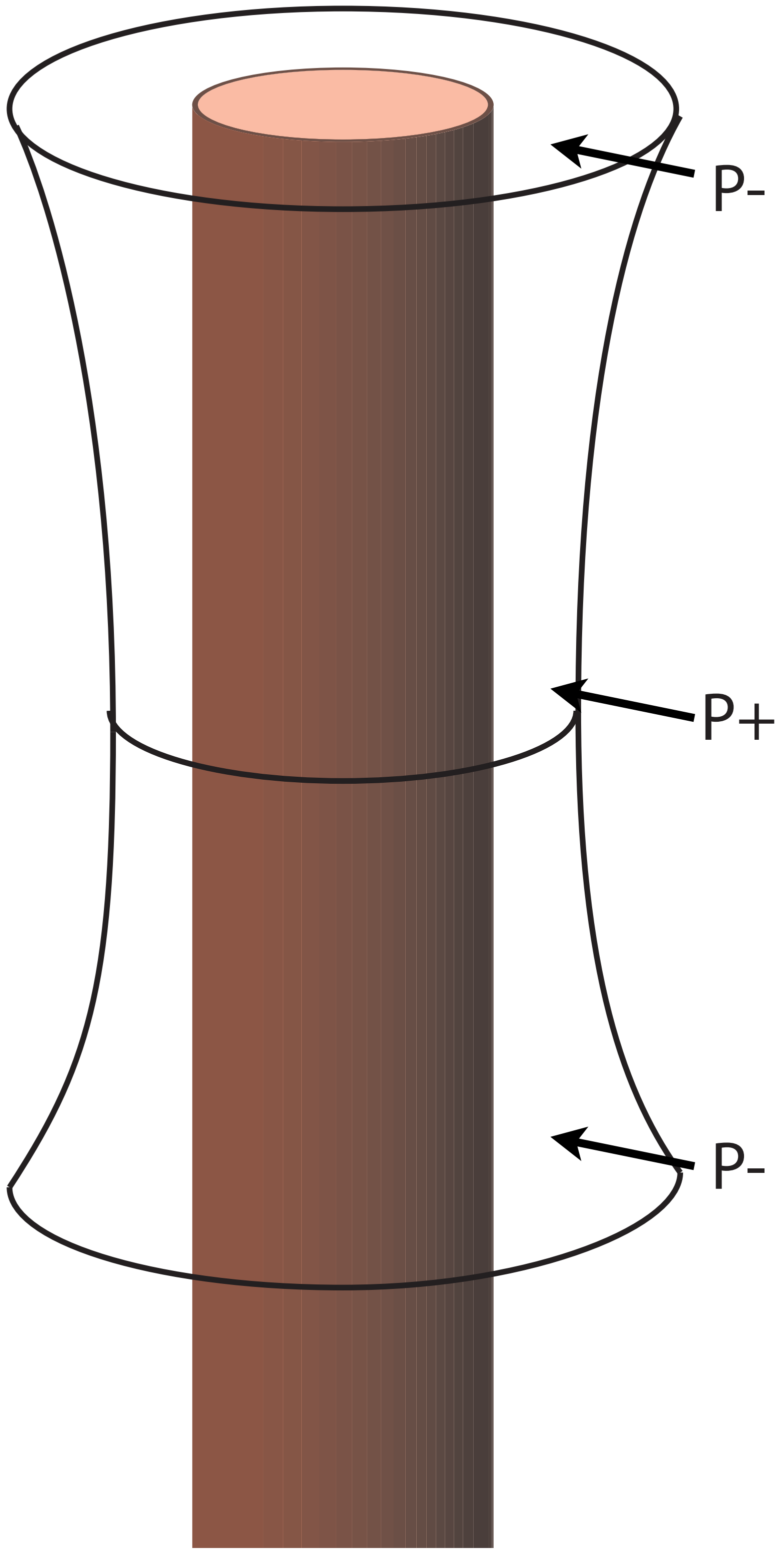




Pressure inside a bubble larger than pressure outside

Pressure in a «plug» lower than outside

# Liquid cylinder: Pressure approach

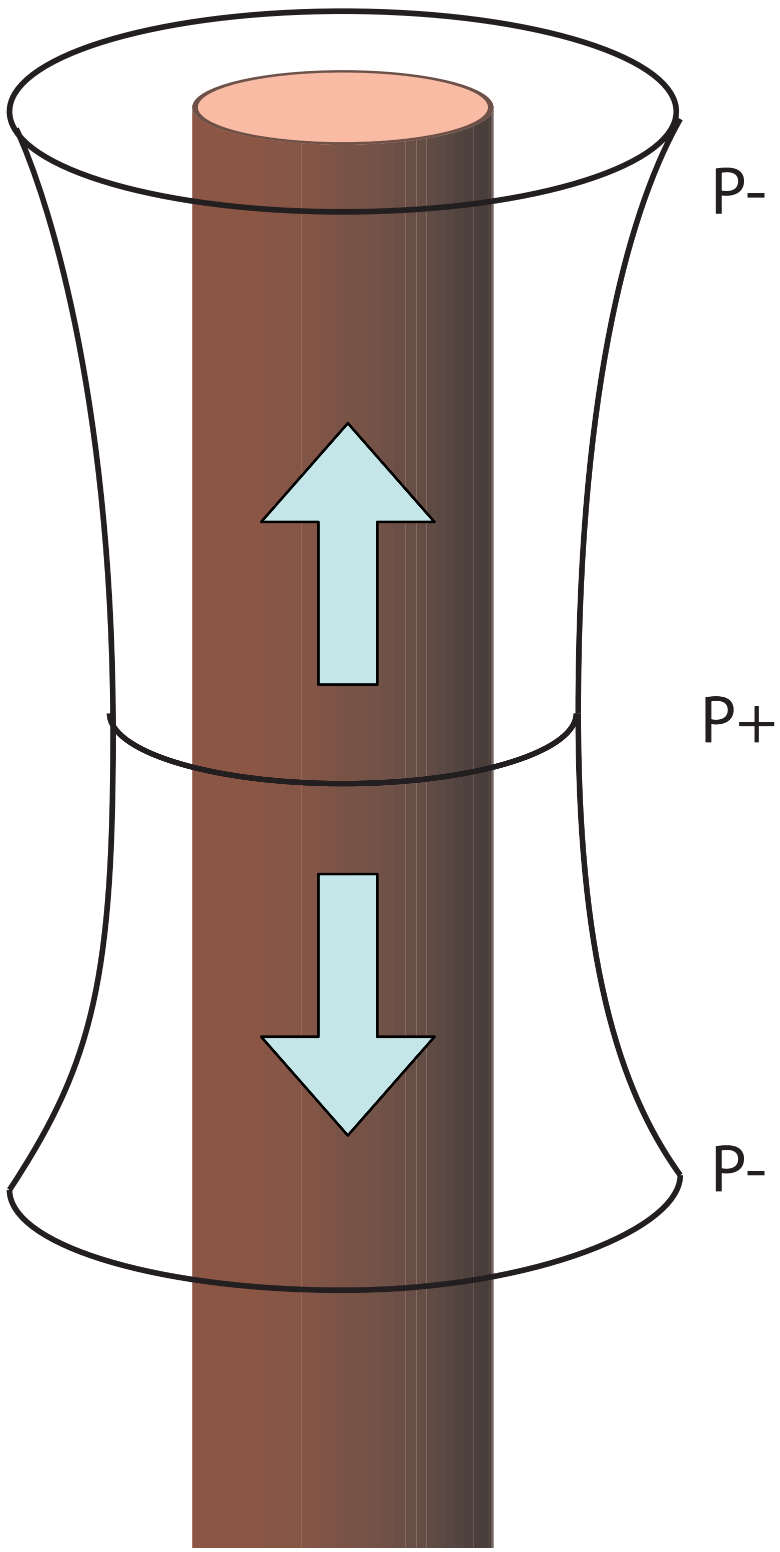
$$\Delta P_{cap} = \gamma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$




What happens if the diameter is perturbed?

# Liquid cylinder

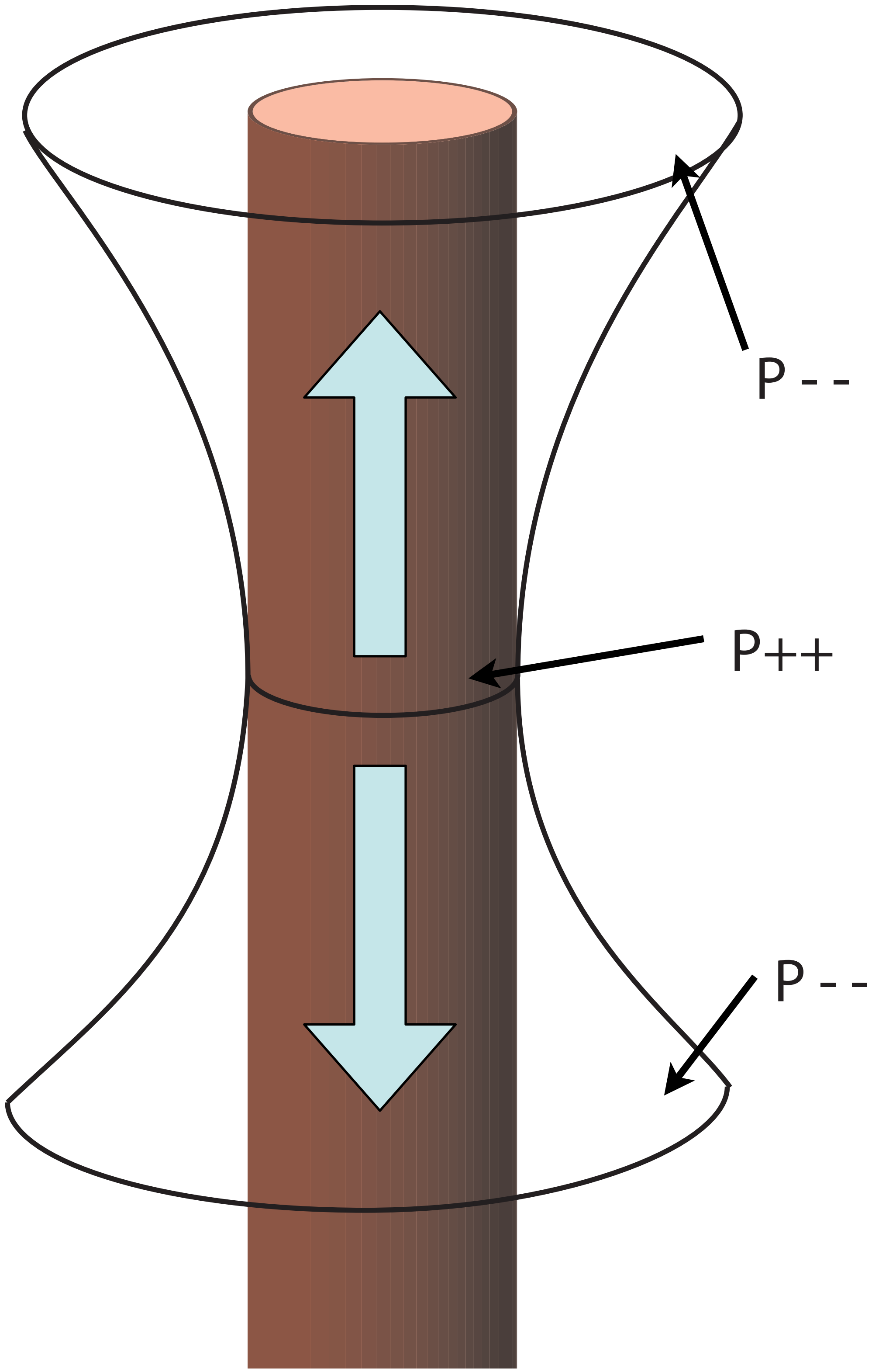
$$\Delta P_{cap} = \gamma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$




A flow takes place from P+ to P-

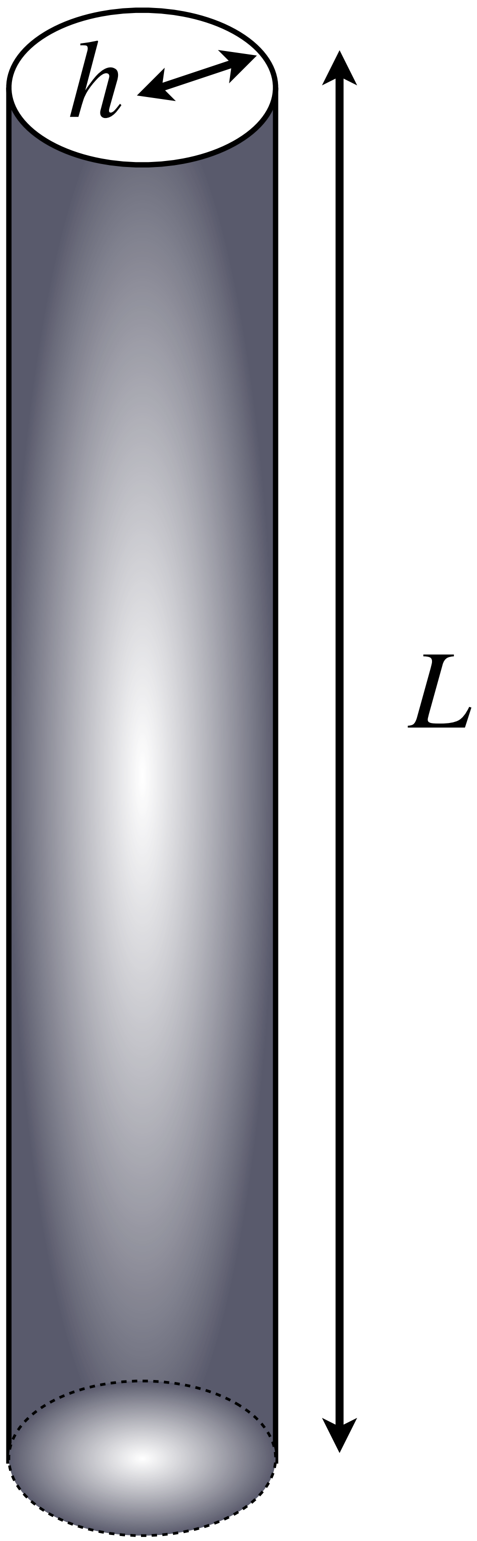
# Rayleigh-Plateau instability

$$\Delta P_{cap} = \gamma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$




Which increases imbalance:
→ Instability

# Energy approach



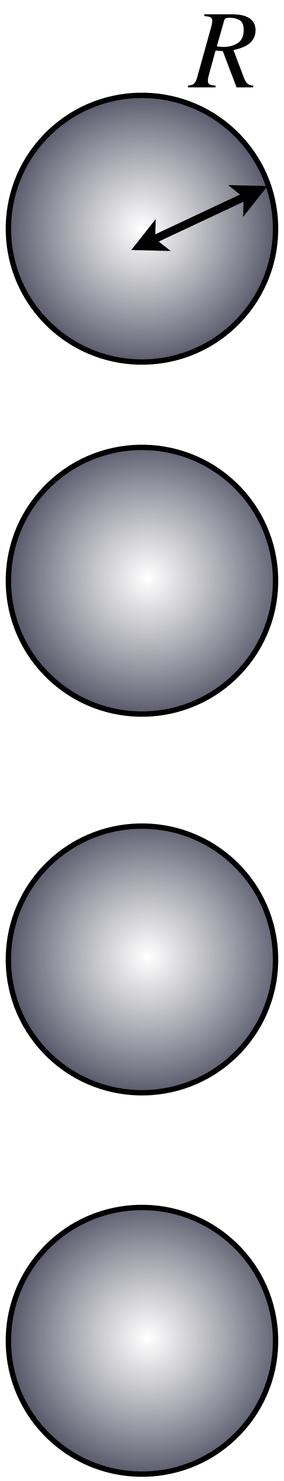

## Jet volume

$$V_{jet} = \pi h^2 L$$

## Surface area

$$A_{jet} = 2\pi h L$$

# Energy approach



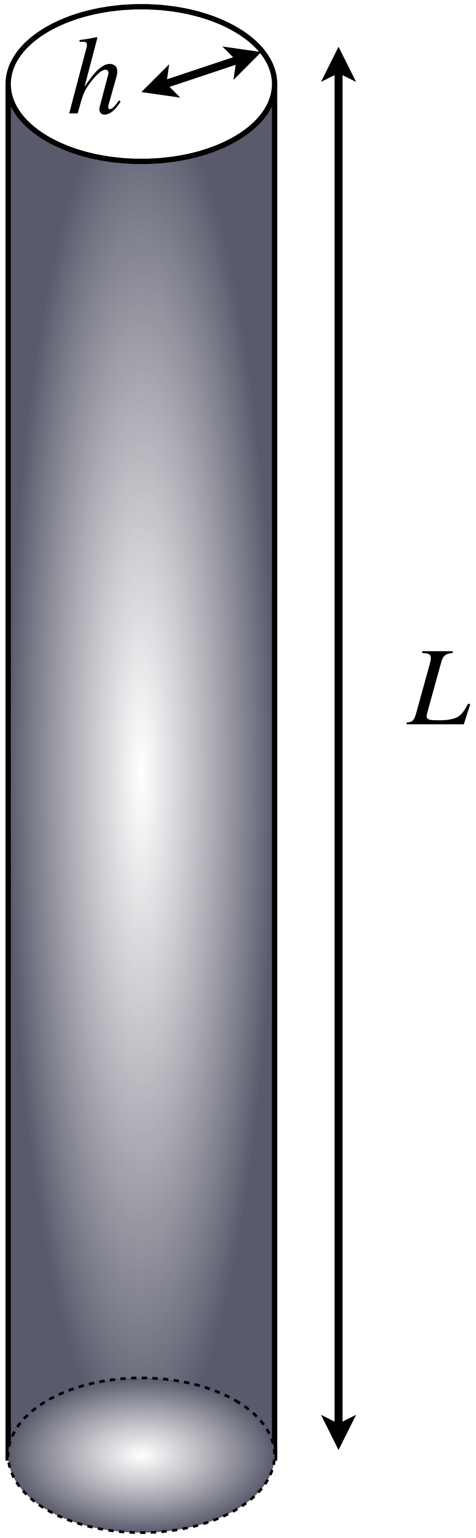

Drop volume

$$V_{drops} = N \cdot \frac{4}{3}\pi R^3$$

Surface area

$$A_{drops} = N \cdot 4\pi R^2$$

# Minimize energy



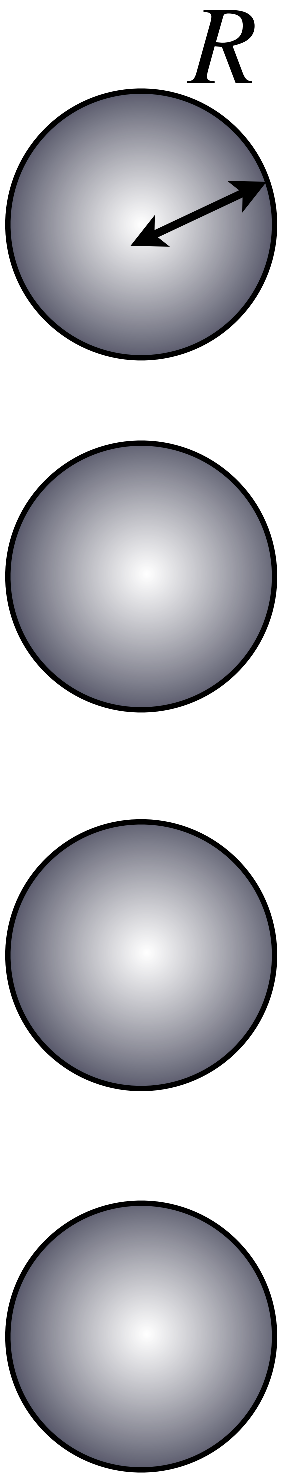

Conserve volume,
compare areas

$$R^3 = \frac{3L}{4N}h^2$$

$$\frac{A_{jet}}{A_{drops}} = \frac{2R}{3h}$$

Surface area is reduced if:

$$R^* > 3h/2.$$

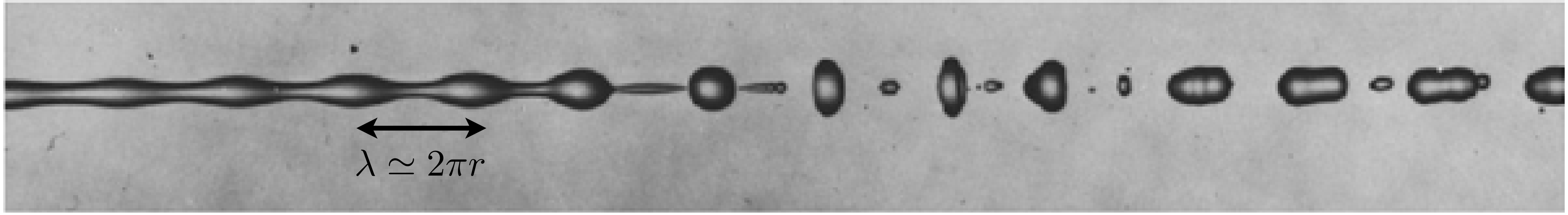


# Classic problem

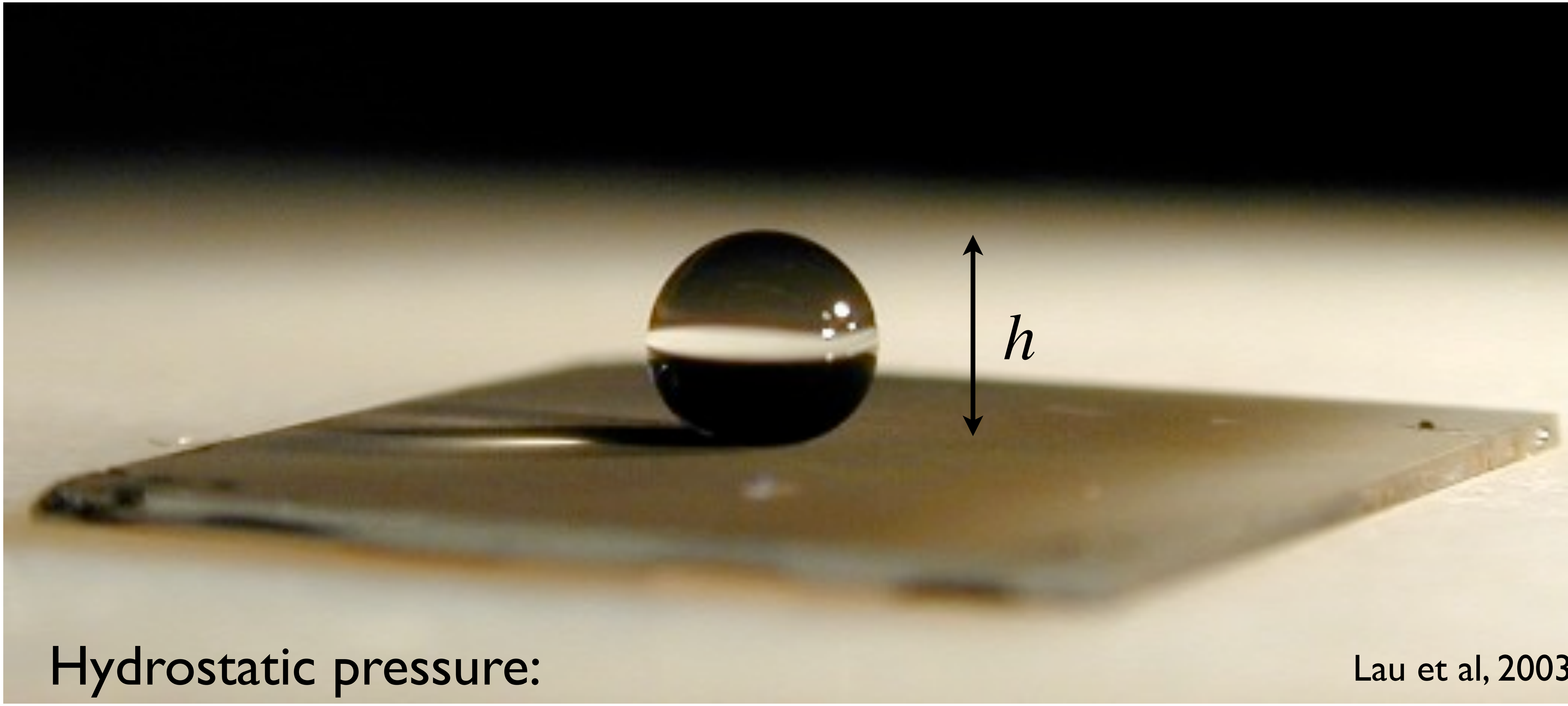
- Plateau (1850) presented this argument and predicted «optimal wavelength» to be

$$\lambda = 2\pi h.$$

- Rayleigh (1879) realized that dynamics must play a role in wavelength selection of $\lambda > \lambda cr$
- He found $\lambda = 9h$



# Capillary length




Lau et al, 2003

Hydrostatic pressure:

$$P_{hs} \sim \rho g h$$

Capillary pressure:

$$P_{cap} \sim \frac{\gamma}{h}$$

$$P_{hs} = P_{cap} \Rightarrow \boxed{L_C \sim \sqrt{\frac{\gamma}{\rho g}}}$$

# In micro-systems

Size always smaller than $L_C$

→ Gravity can always be ignored

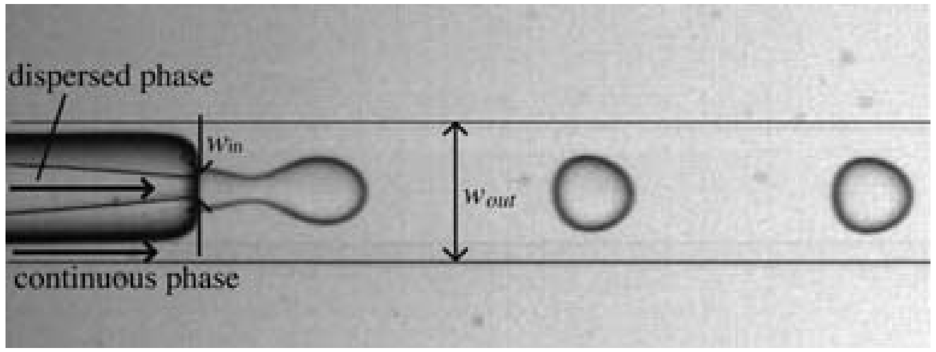
Inertial effects can also be ignored

Viscous vs. Capillary competition

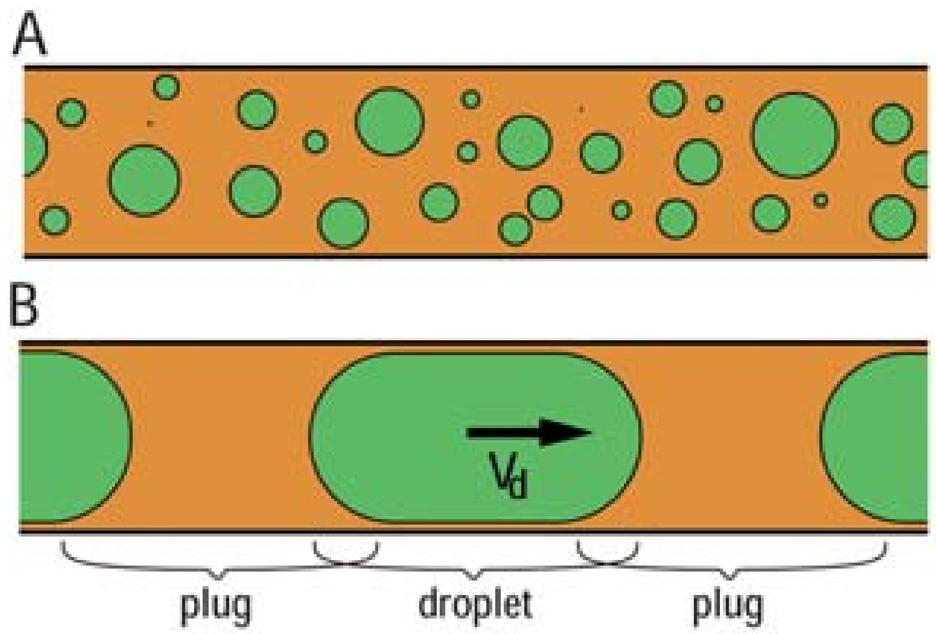
$$Ca = \frac{\mu U}{\gamma}$$

For a given fluid pair, *Ca* is a measure of the velocity of flow only

# Multiphase flows in micro-channels

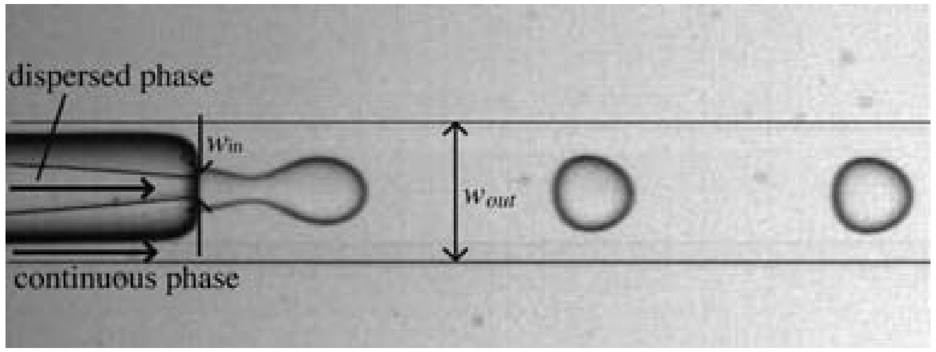



Drop production




Transport

# Production



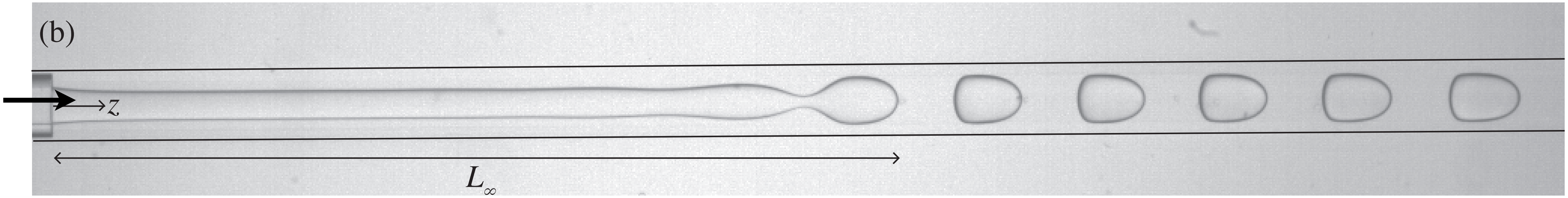

Drop production

# In a flowing, confined system

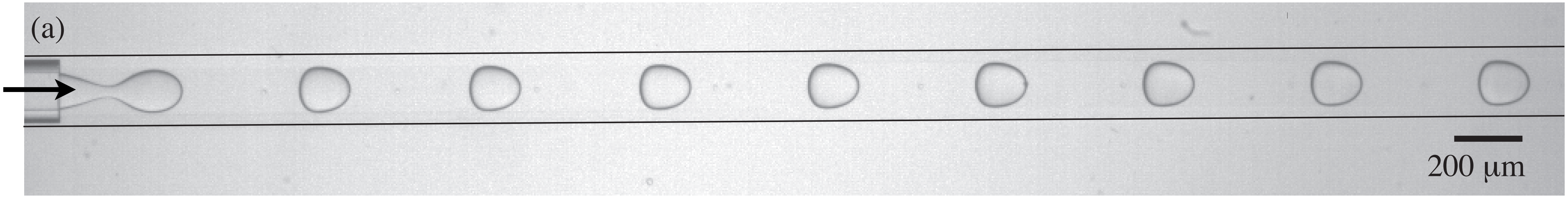
*Qext=30 µL/min*

*Qint=14 µL/min*

jetting




dripping



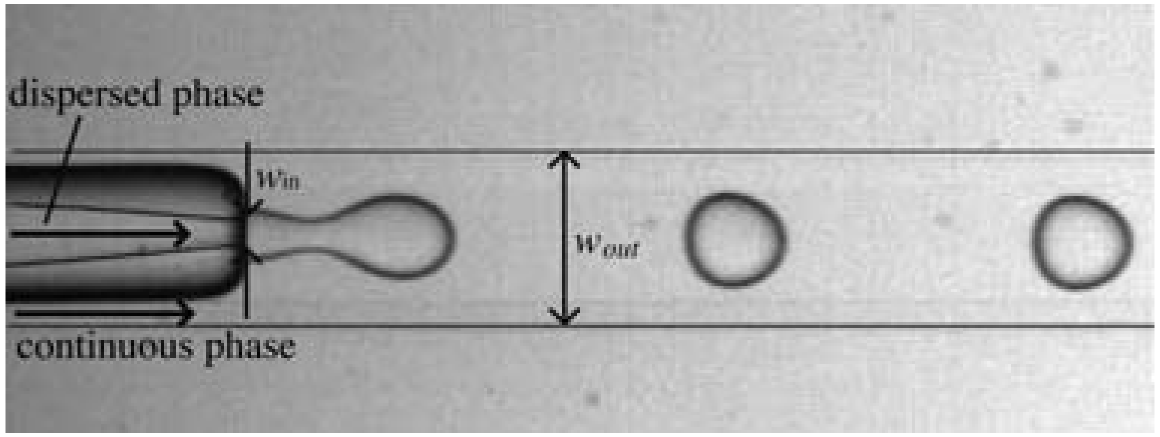

*Qint=5 µL/min*

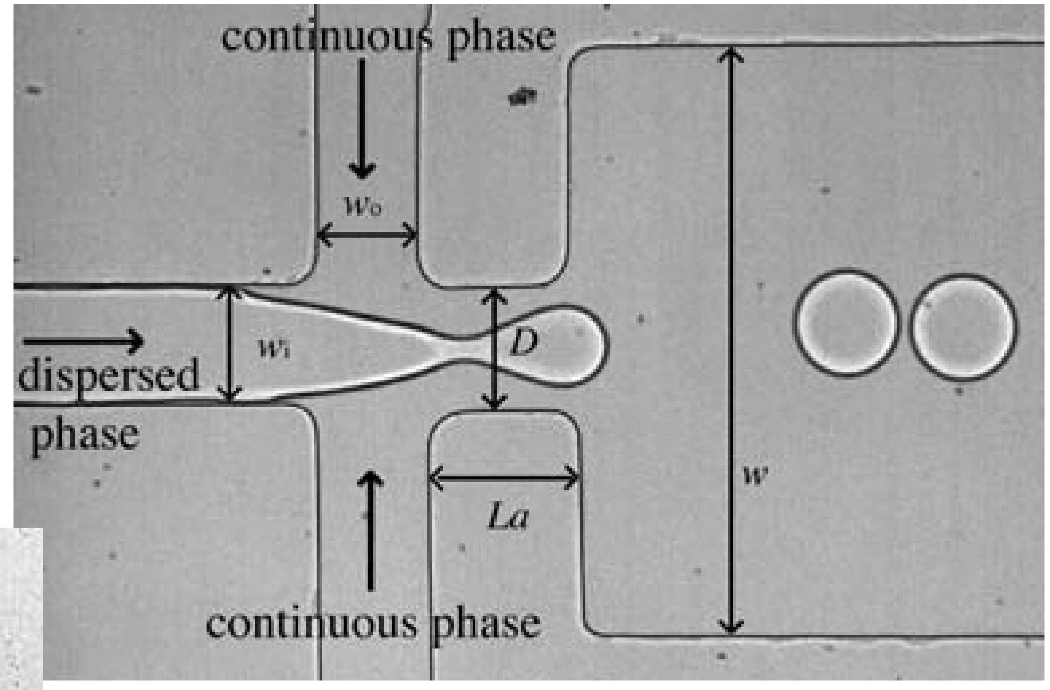
Cordero & Baroud, 2010

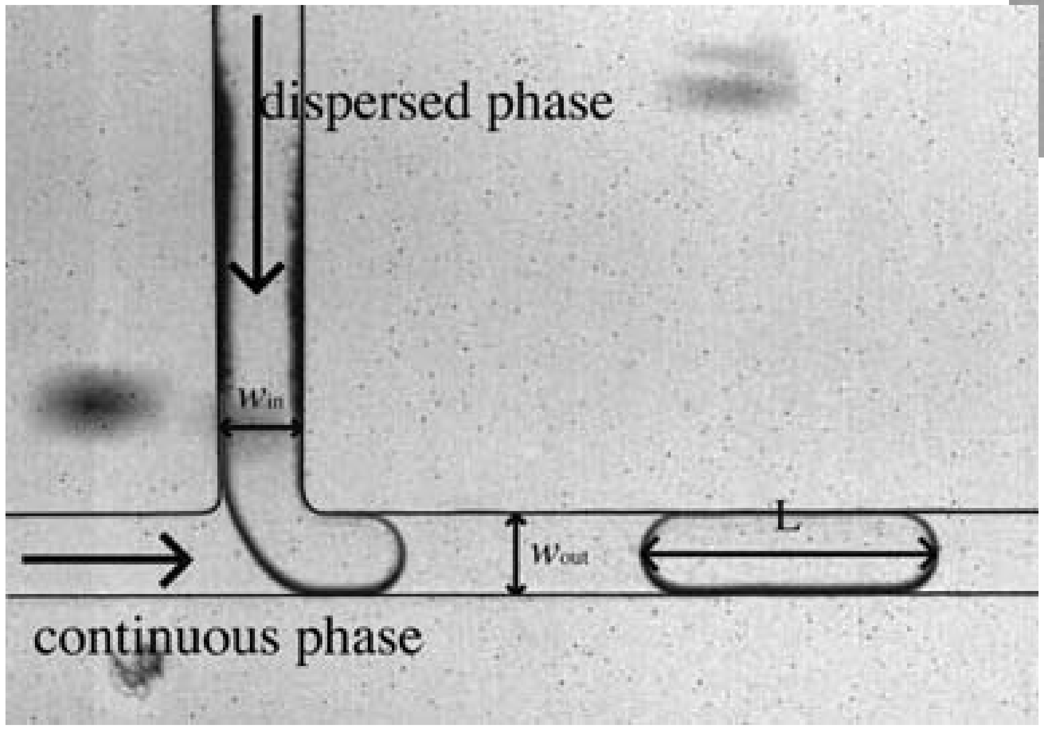
# In microchannels

## Bad news 2: Formation dynamics essentially determined by the geometry

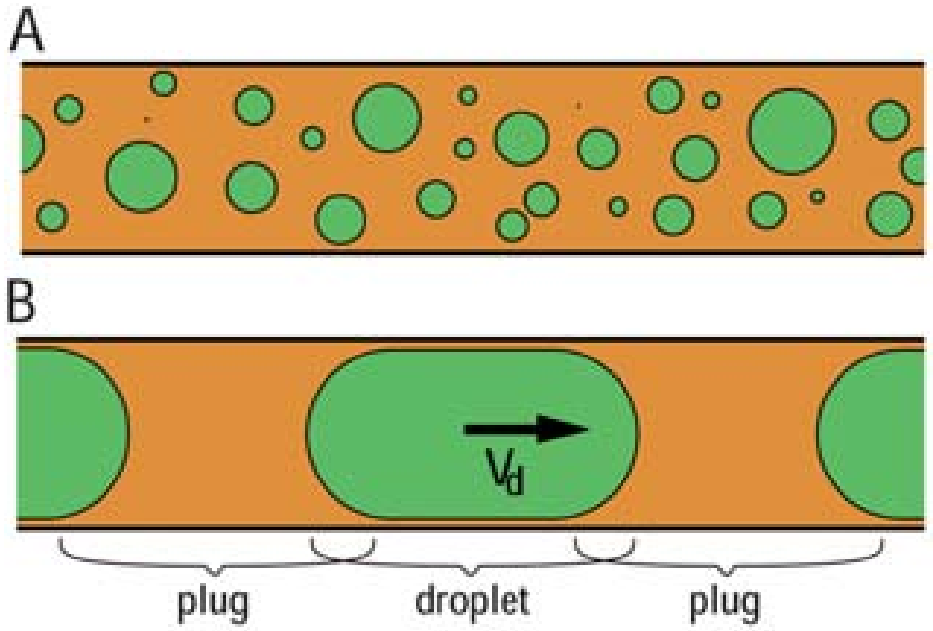
Three classic designs are now standard



# Multiphase flows in micro-channels

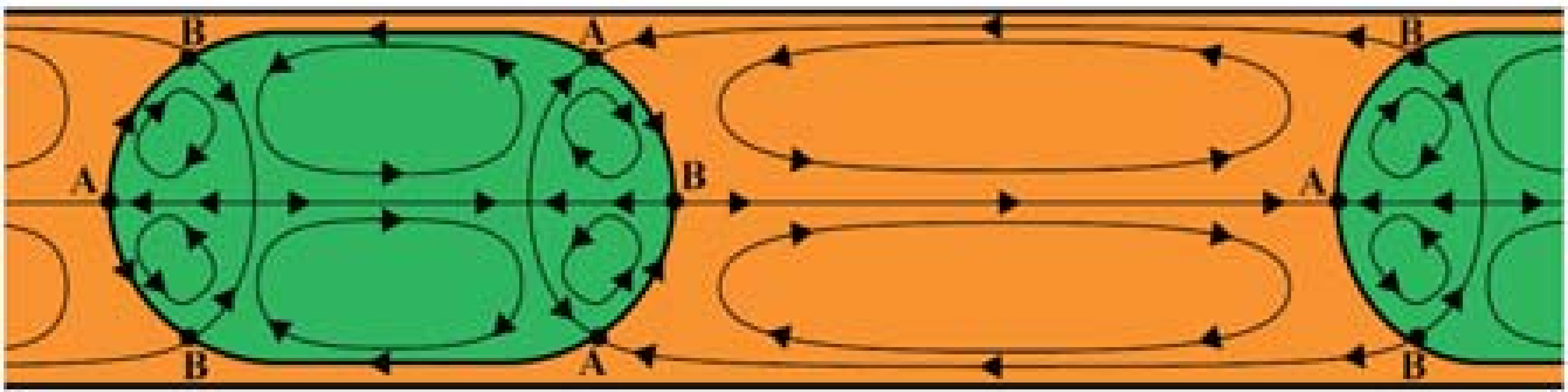


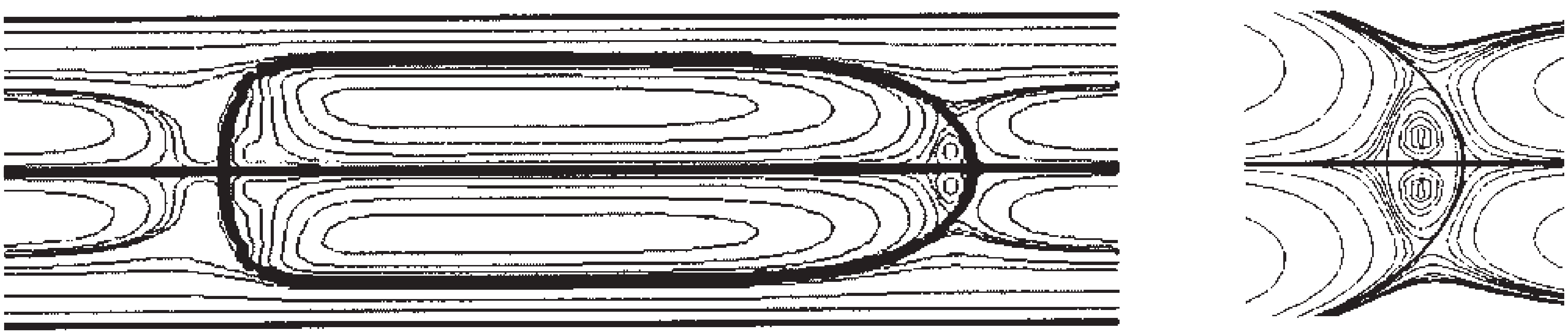

Transport

How do big drops flow in a microchannel?

# Drop creates recirculation

$\lambda << 1$



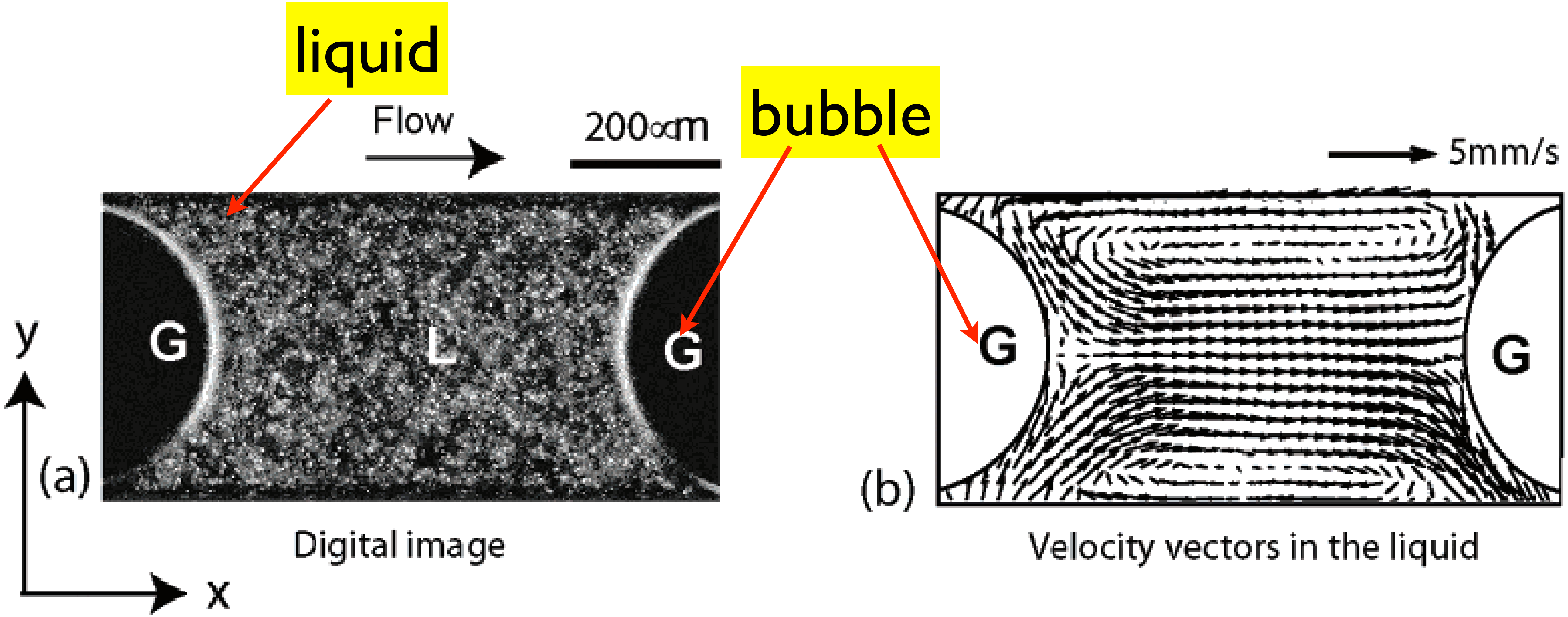
(a)

(b)

2D computations

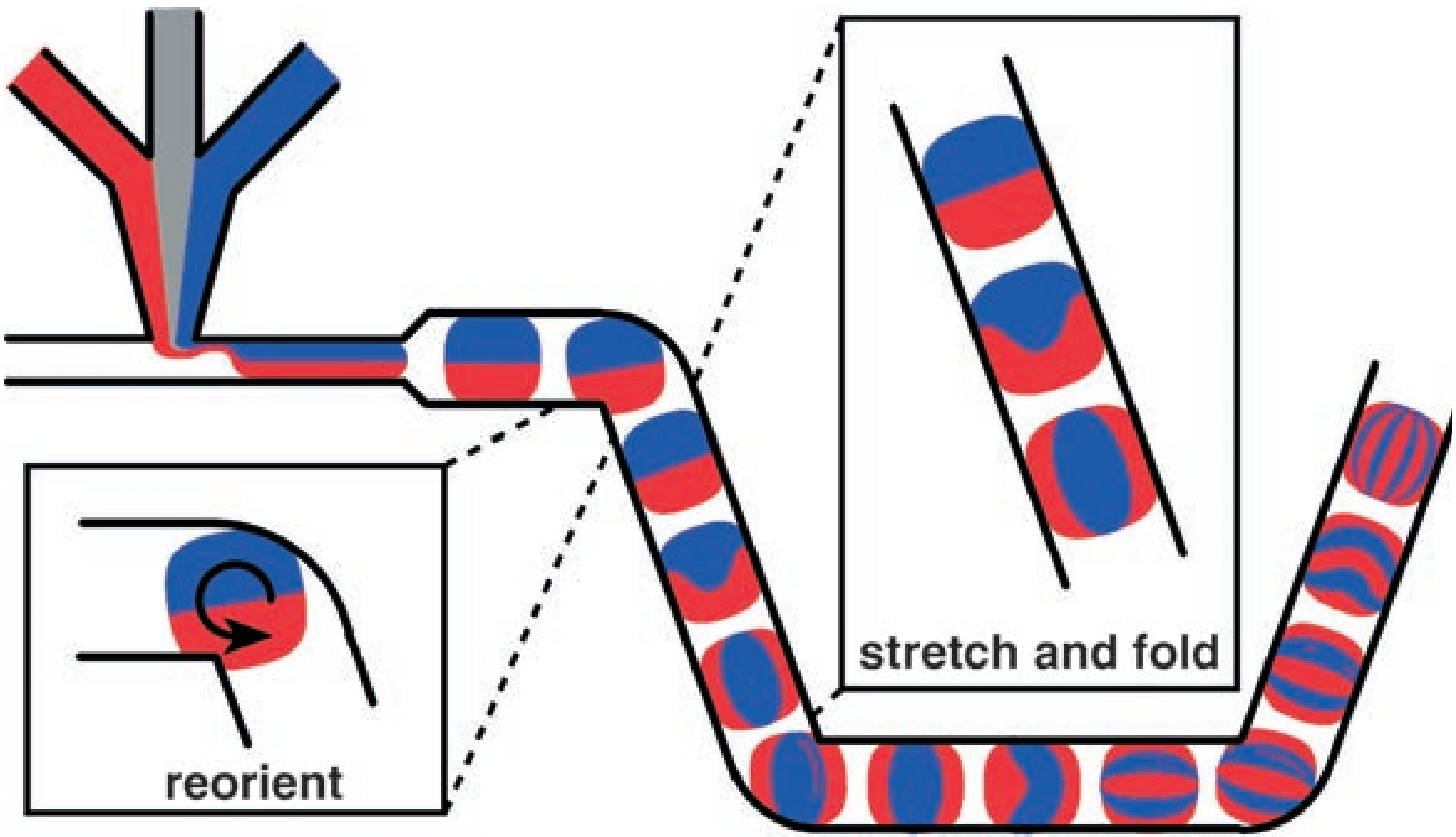
Sarrazin et al, 2006

# Recirculating flows



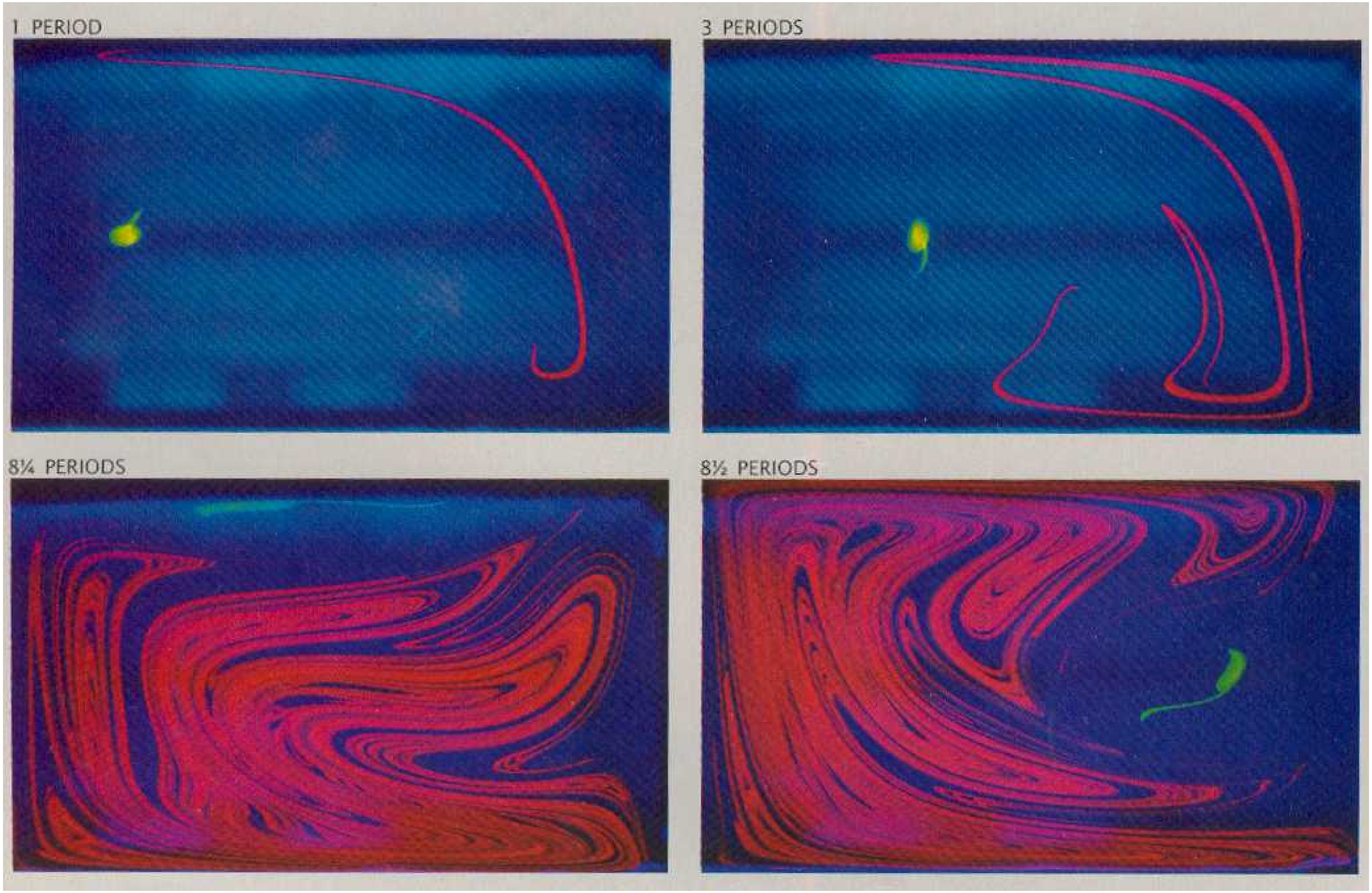

Jensen lab, MIT, 2005

# Can we use this to mix fluids?



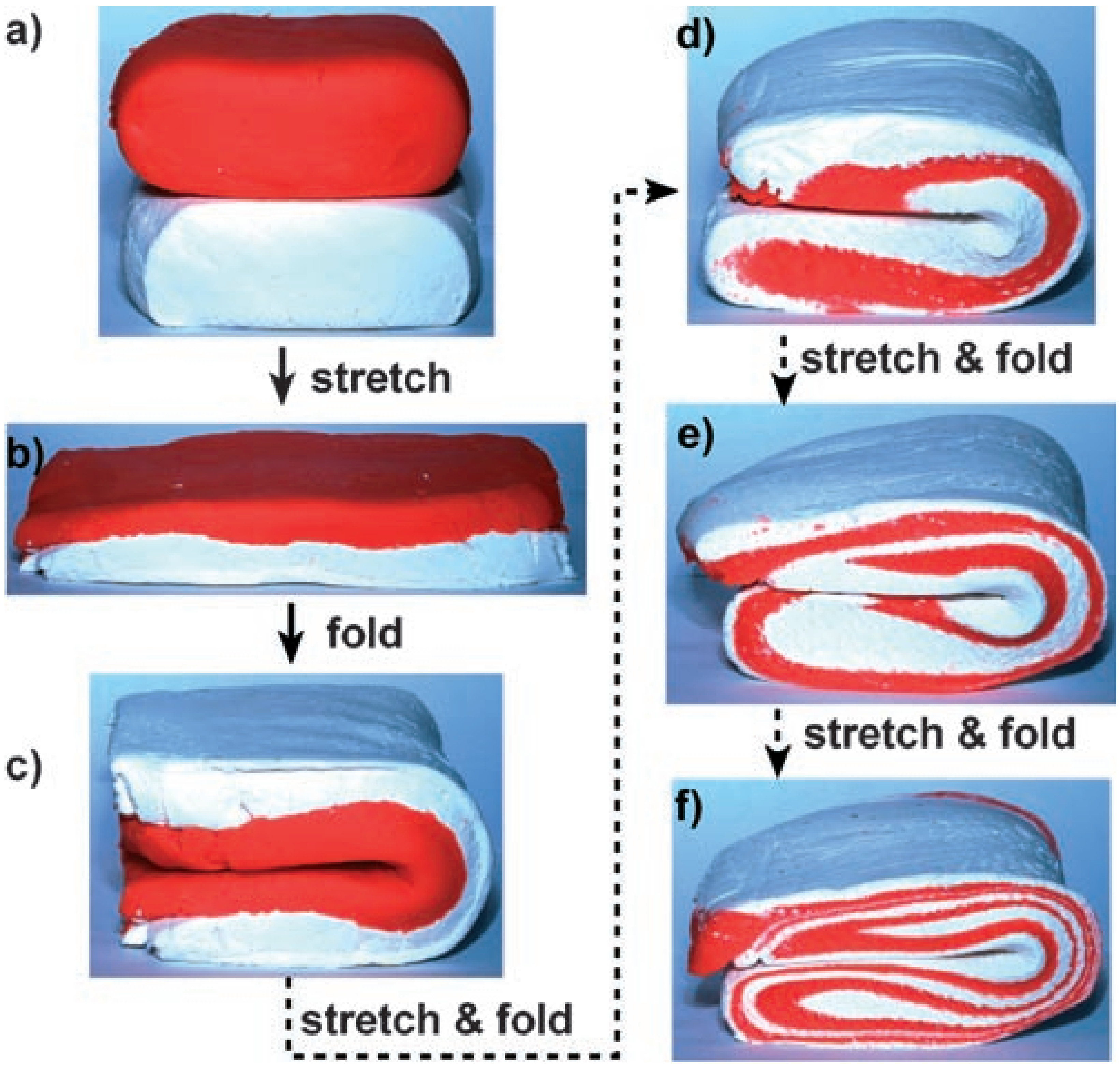

Song et al, 2003

# Chaotic mixing



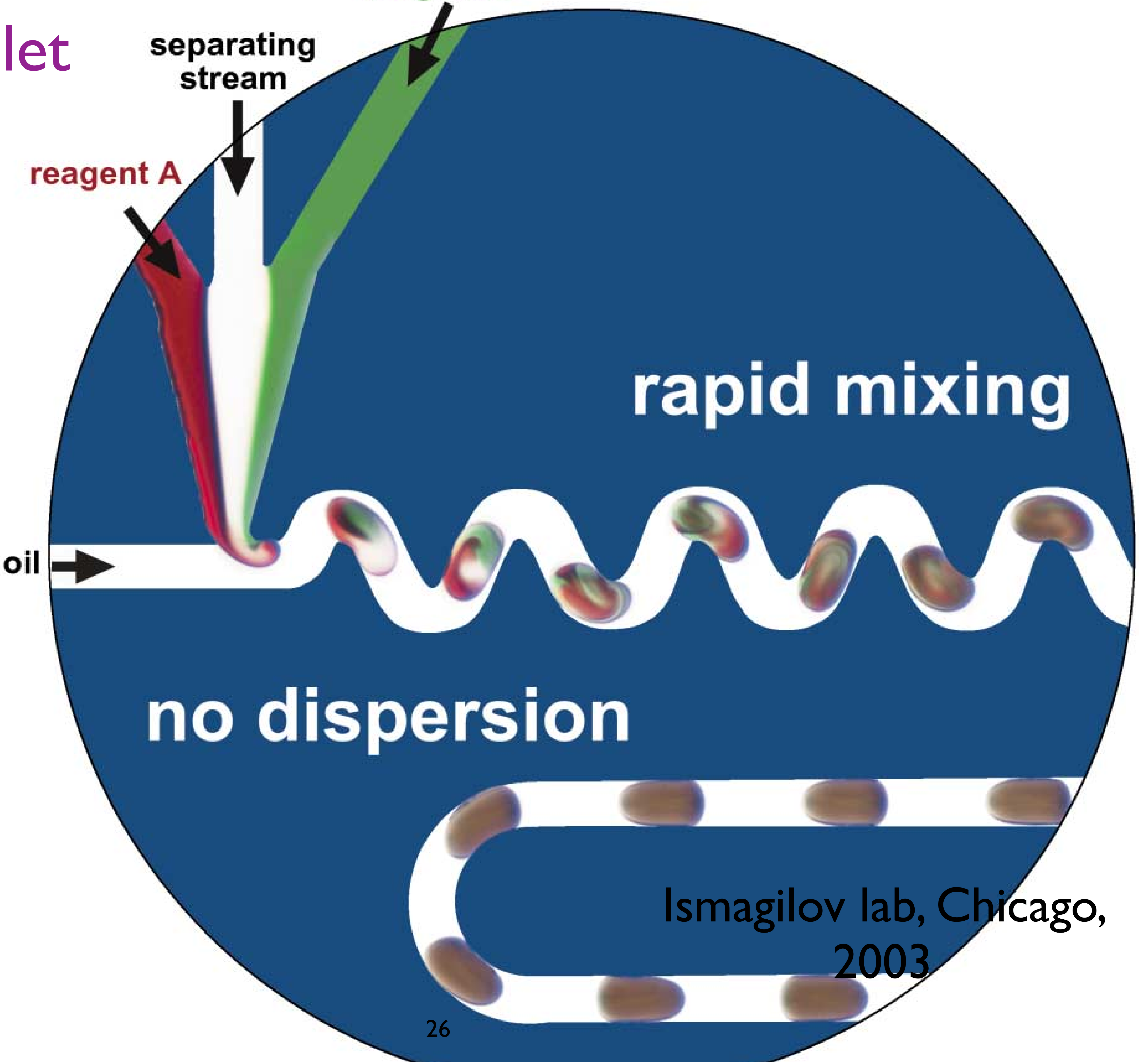

Ottino

# Chaotic mixing (modern)



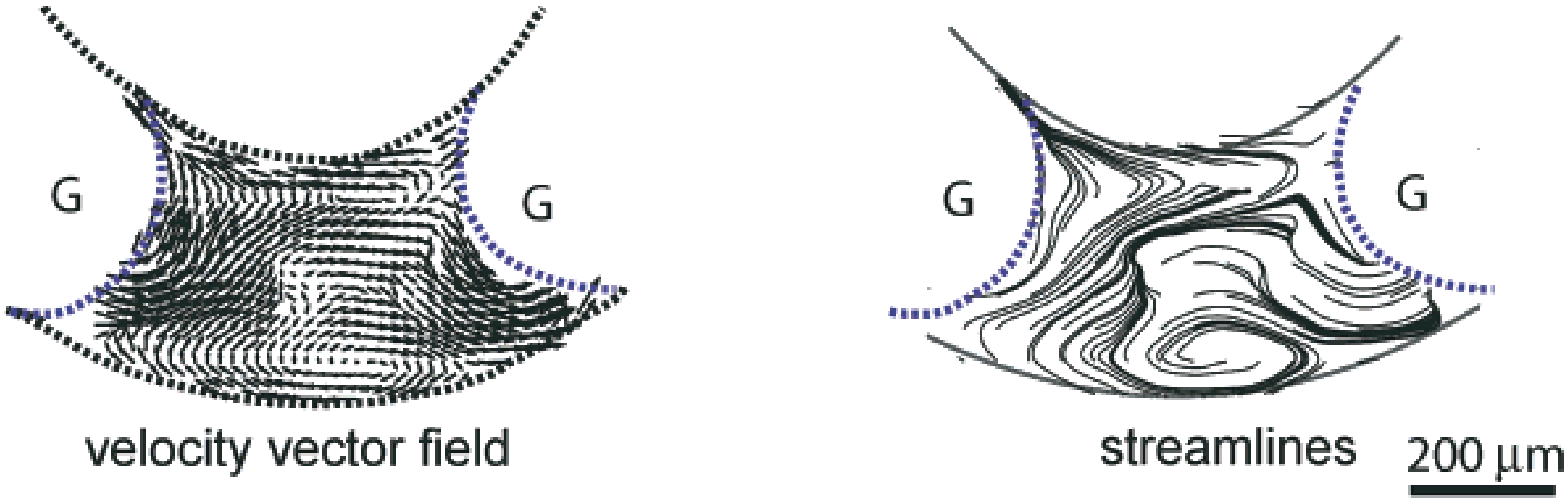

Song et al, 2003

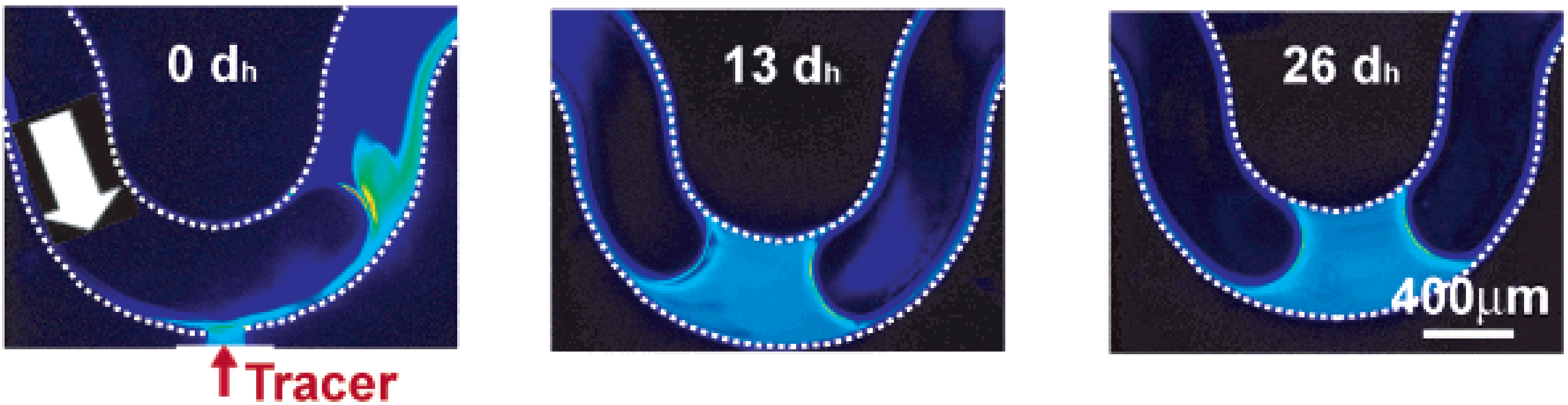
# In a droplet




Ismagilov lab, Chicago, 2003

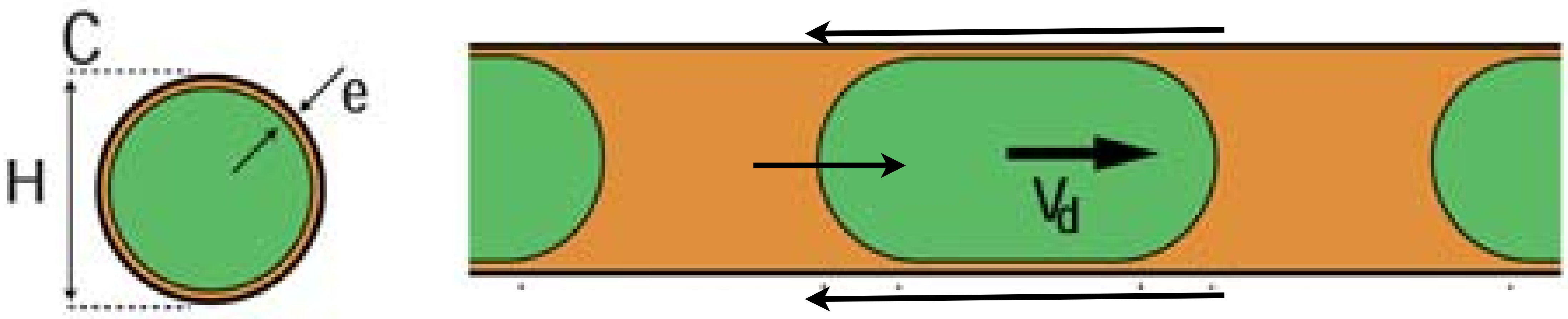
# In a plug




Jensen lab, MIT, 2005

# How fast do drops flow?




Backward flow in film pushes drop faster than mean velocity

$$\frac{V_d - V_{ext}}{V_d} \propto \mathrm{Ca}_d^{2/3}$$

(Drop goes faster than mean velocity!)

See Fairbrother & Stubbs, 1935

# In a rectangular micro-channel

*The steady propagation of an air finger into a rectangular tube* 181

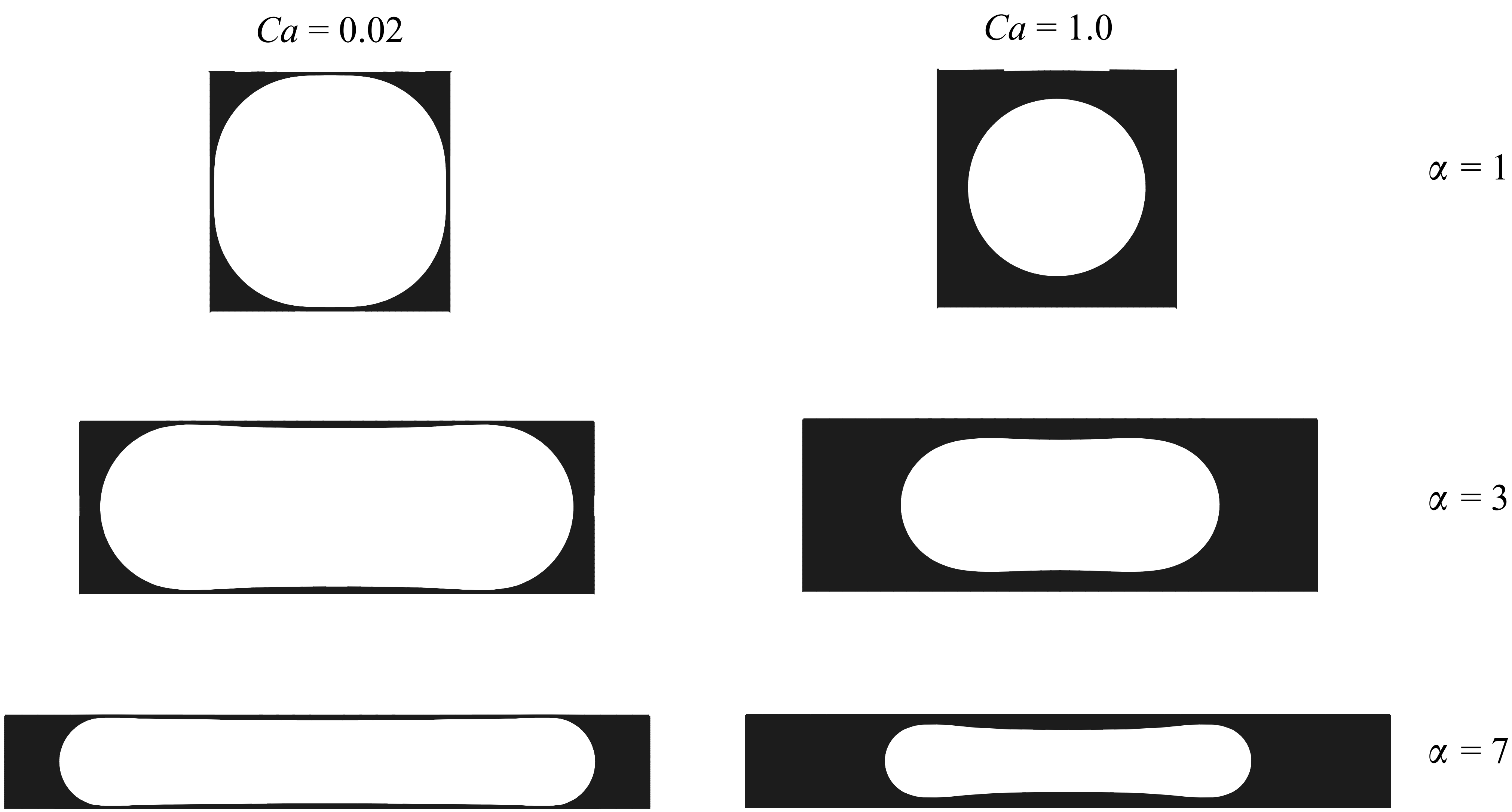
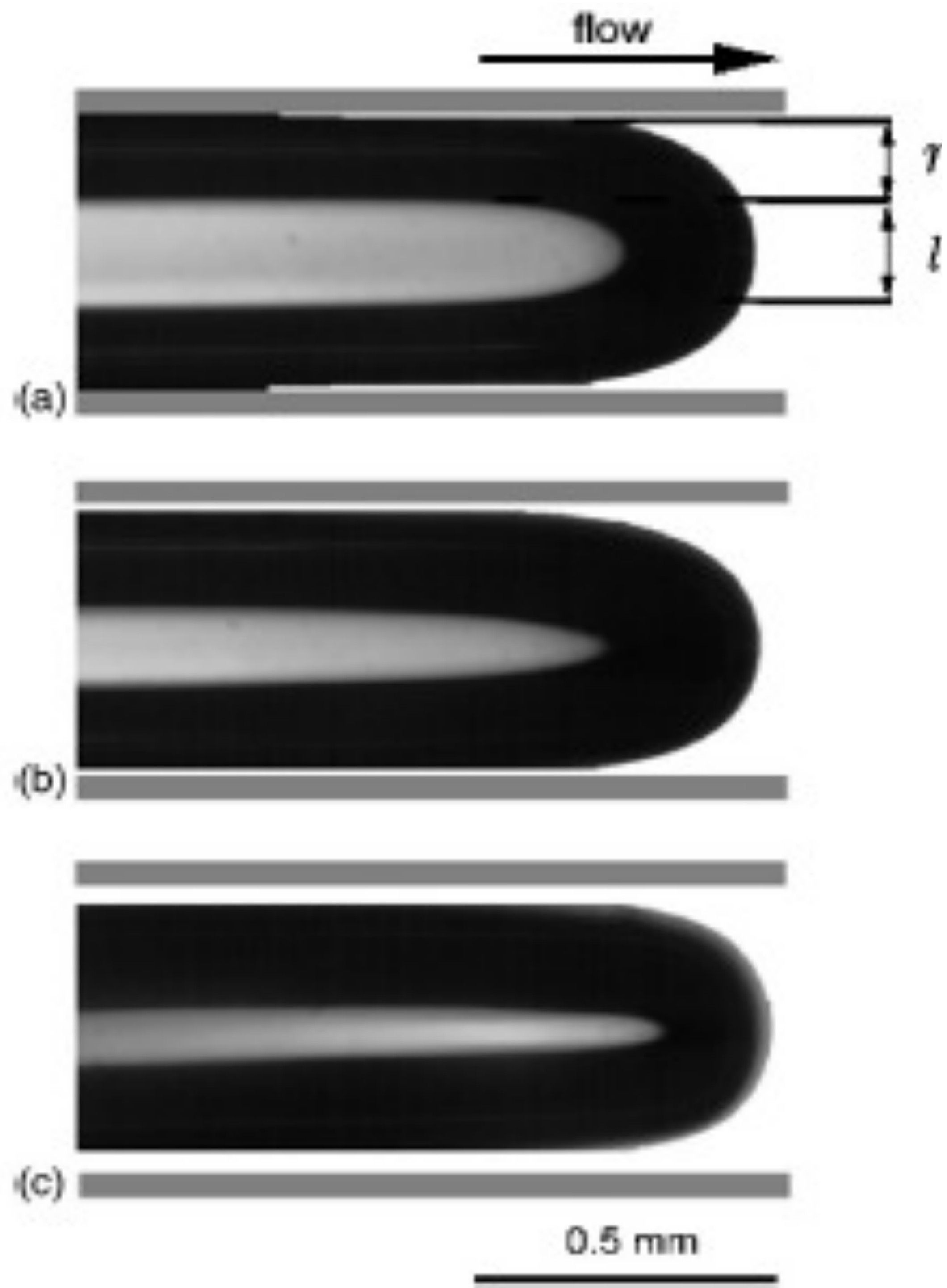



FIGURE 5. Cross-sections of the flow domain for two capillary numbers ($Ca = 0.02, 1.0$), three aspect ratios ($\alpha = 1, 3, 7$) and $Bo = 0$, at a distance of $3.92\alpha$ behind the tip.

deLozar et al. 2008

# Drops leave a film behind them




« Bretherton » problem

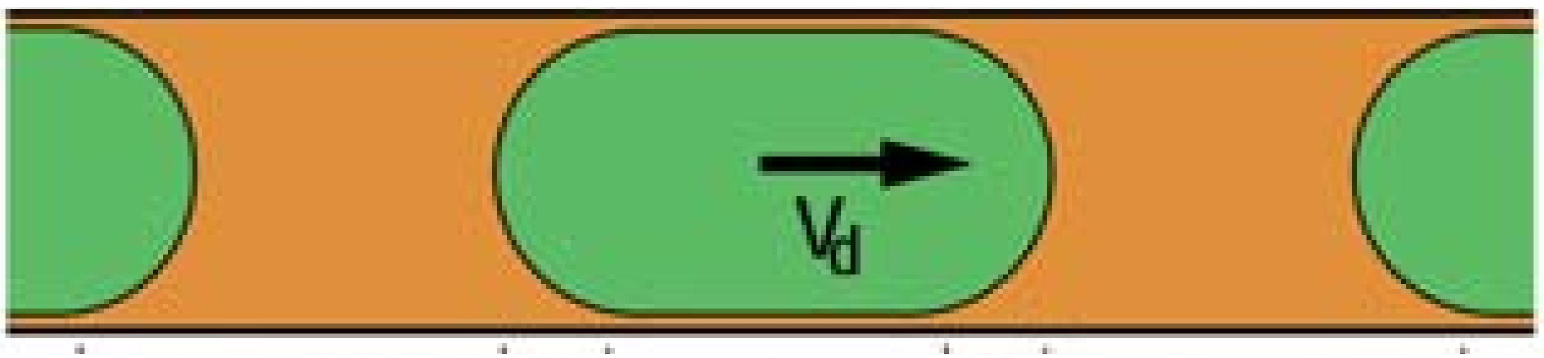
$$\frac{e}{H} \propto \mathrm{Ca}_d^{2/3}.$$

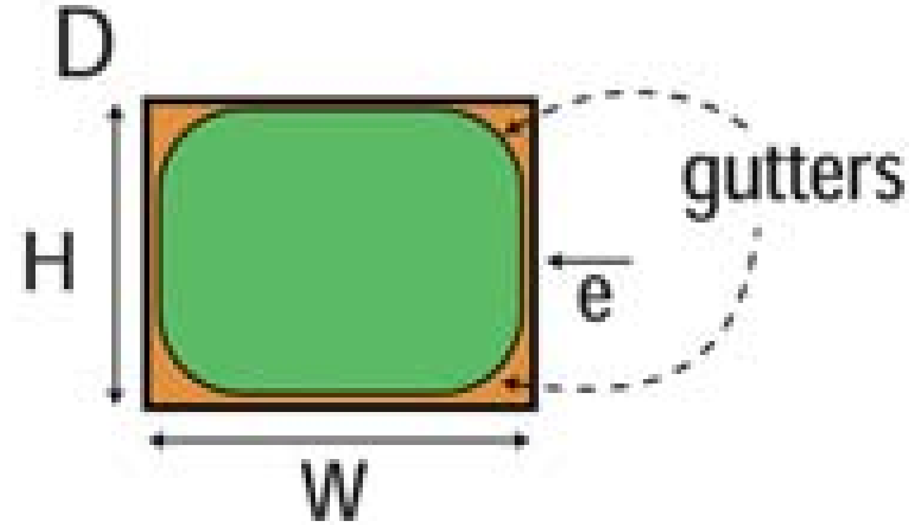
Film thickness increases with increasing Capillary number

(Same physics as Landau-Levich film)

Cubaud & Ho, 2004

# How fast the drops flow 2



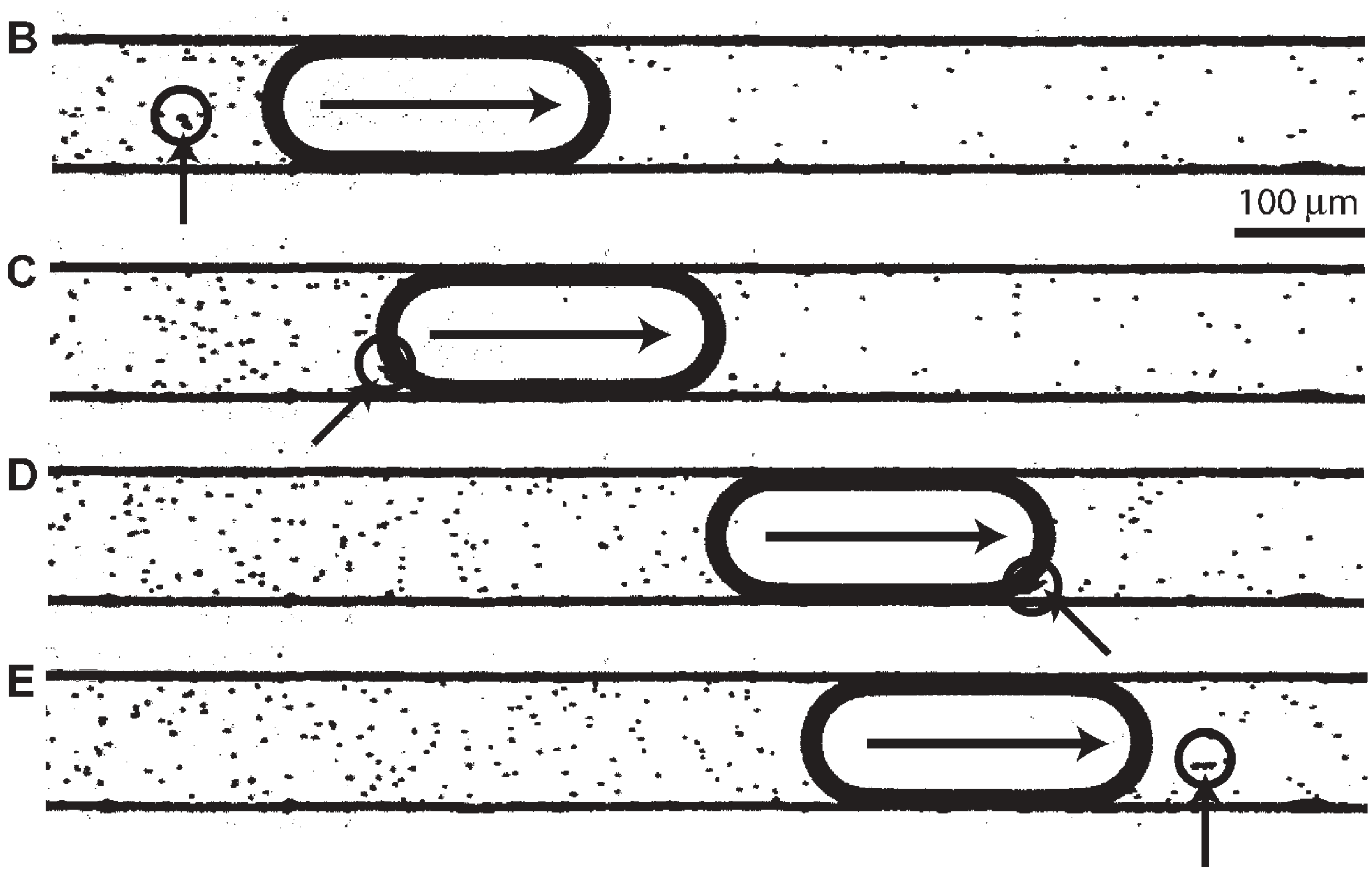

Rectangular channels: Gutters change everything

Flux in gutters directed along Vd, so drops go slower than mean velocity

$$\frac{V_d - V_{ext}}{V_d} \propto -\mathrm{Ca}_d^{-1/3}$$

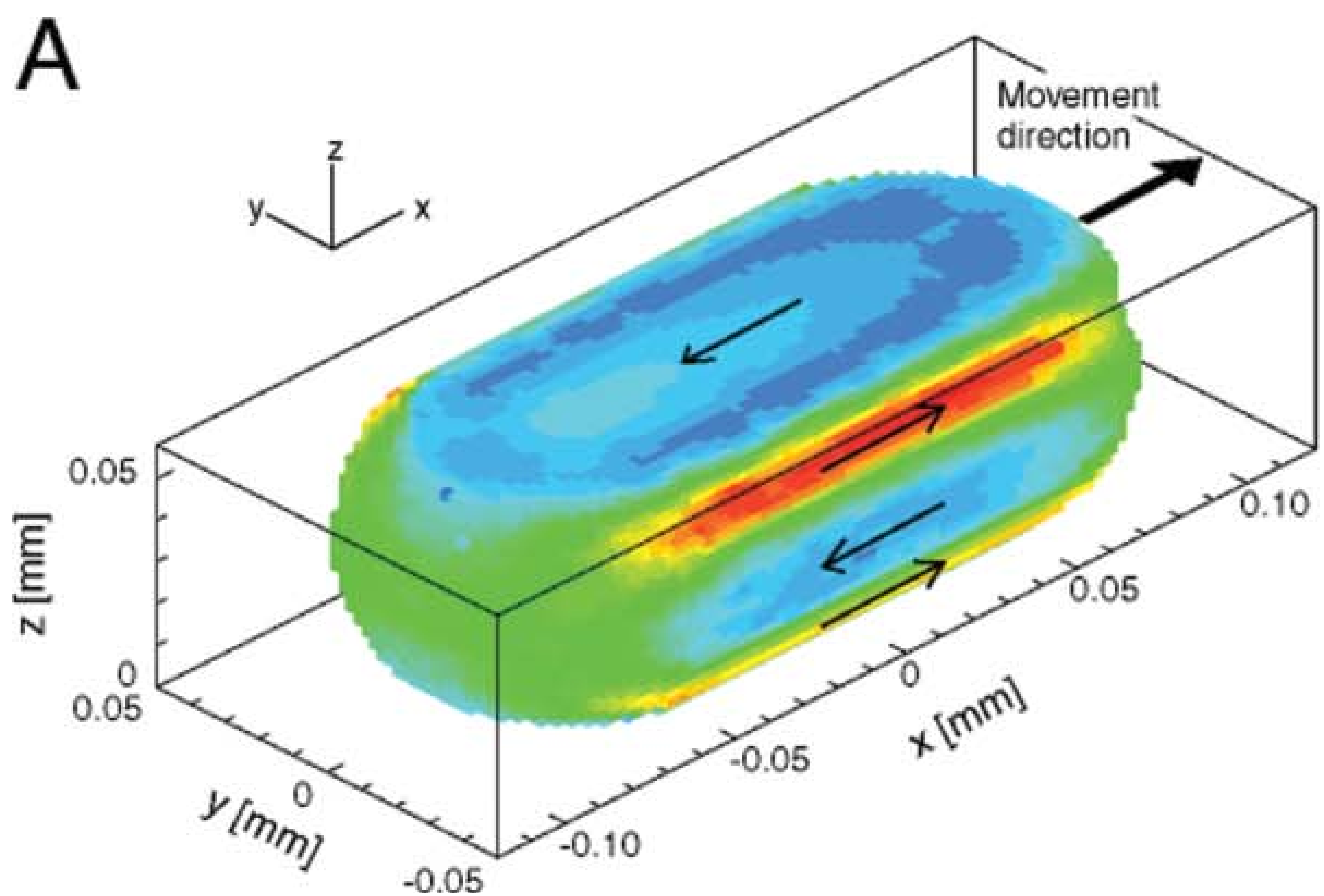
See Wong, Radke, Morris, 1995

# What about in experiments?




Fuerstman et al, 2007

# Detailed measurments

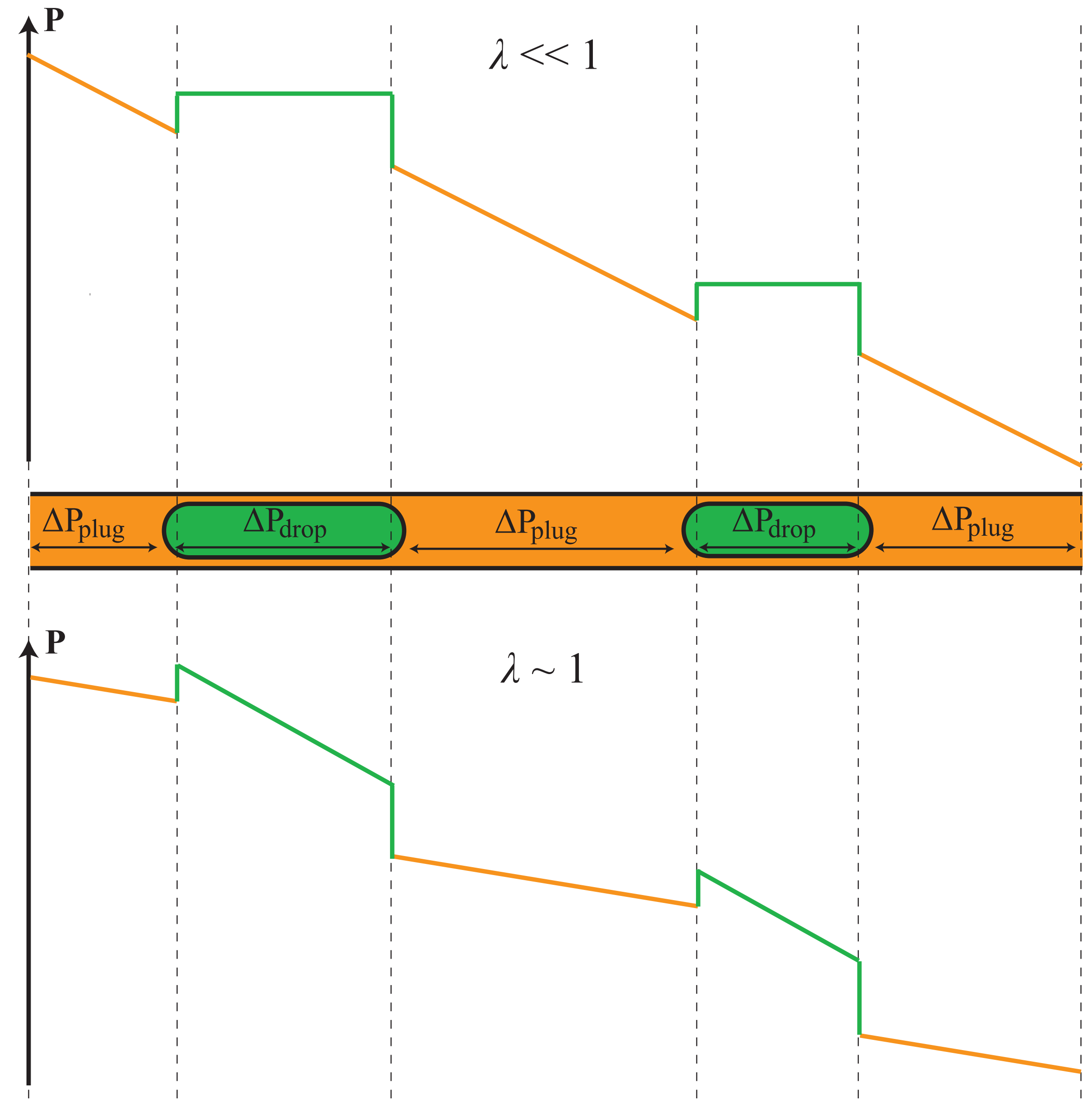



Kinoshita et al, 2007

# Pressure profile in a 2 phase fluid flow

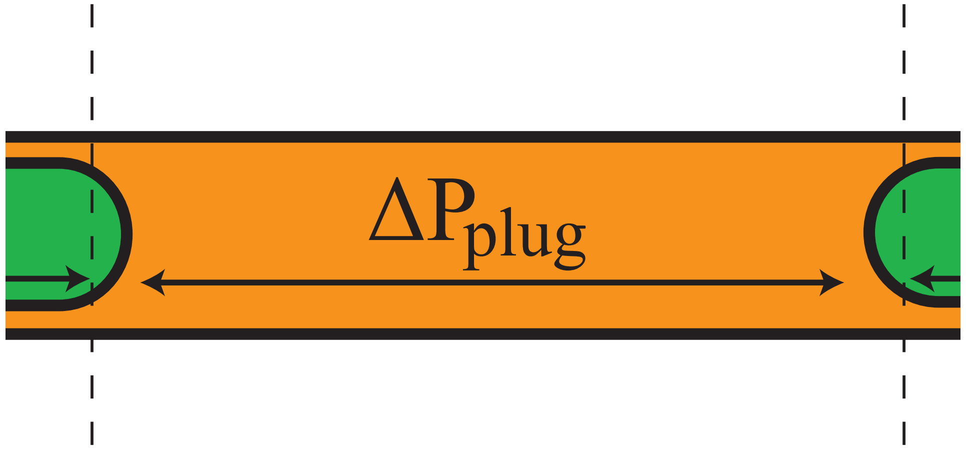
Divide flow into three parts:

- External fluid flow
- Internal fluid flow
- Interfaces



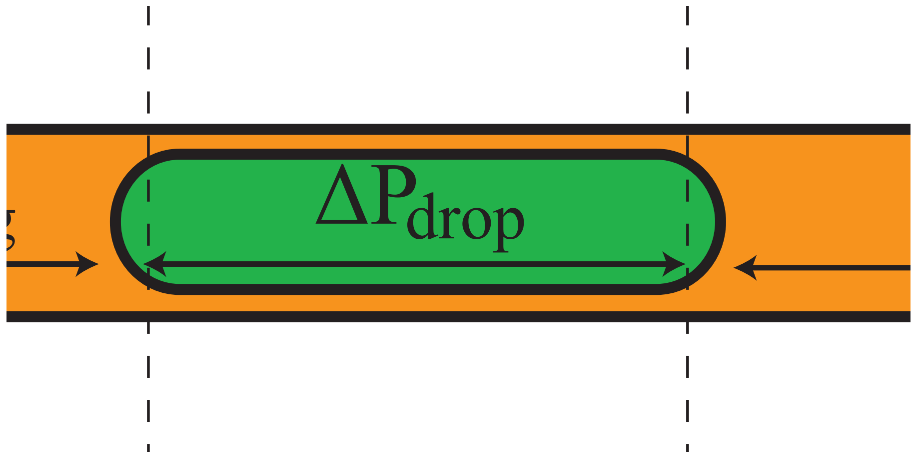
# Flow away from interfaces

Assuming distance between plugs is large, use single phase relation:

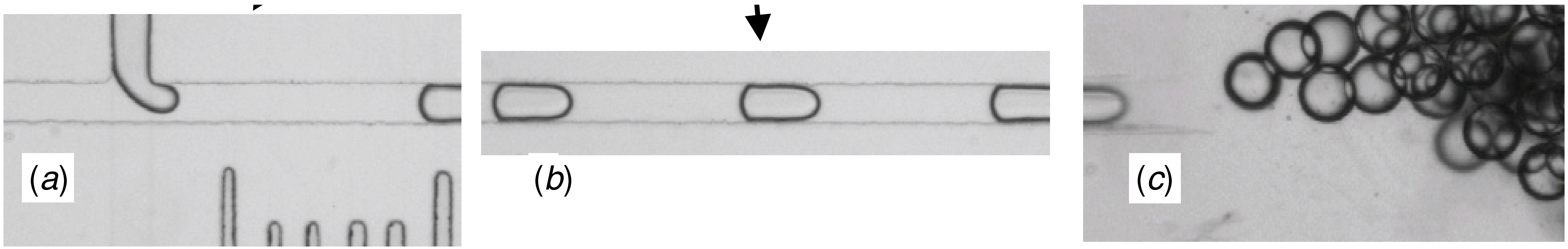


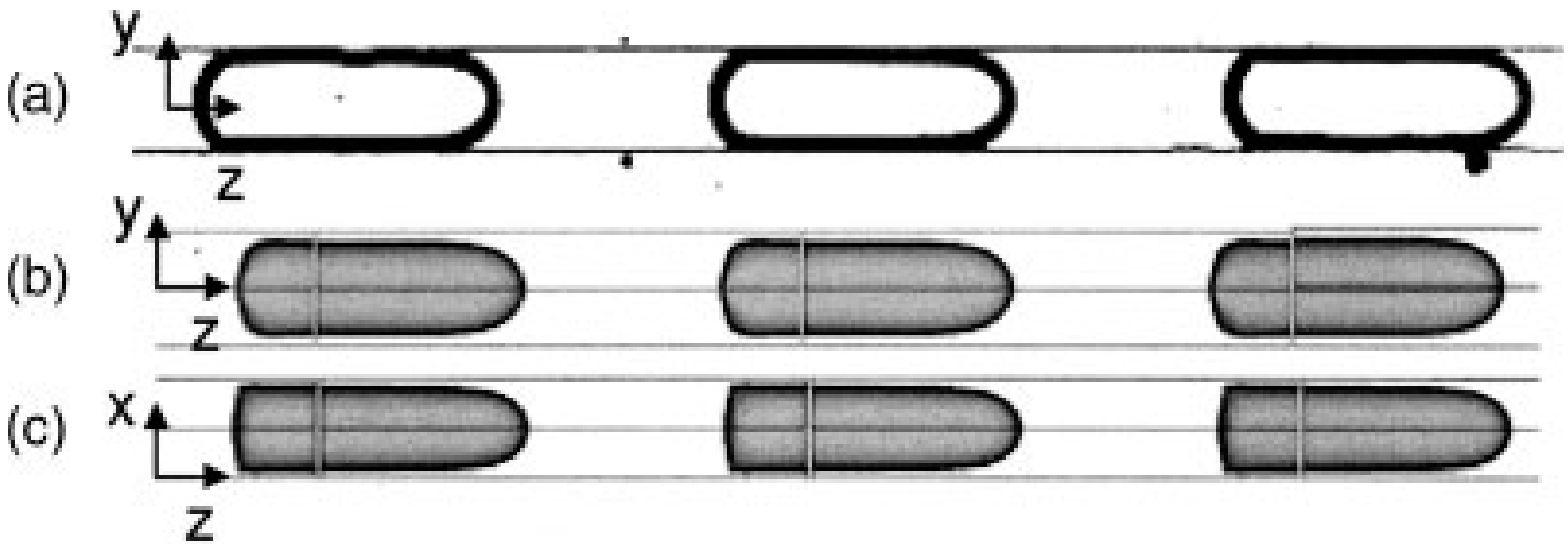

$$\Delta P = \sum_N \mathcal{R}_N Q$$

For each plug and each drop



# Effect of interfaces



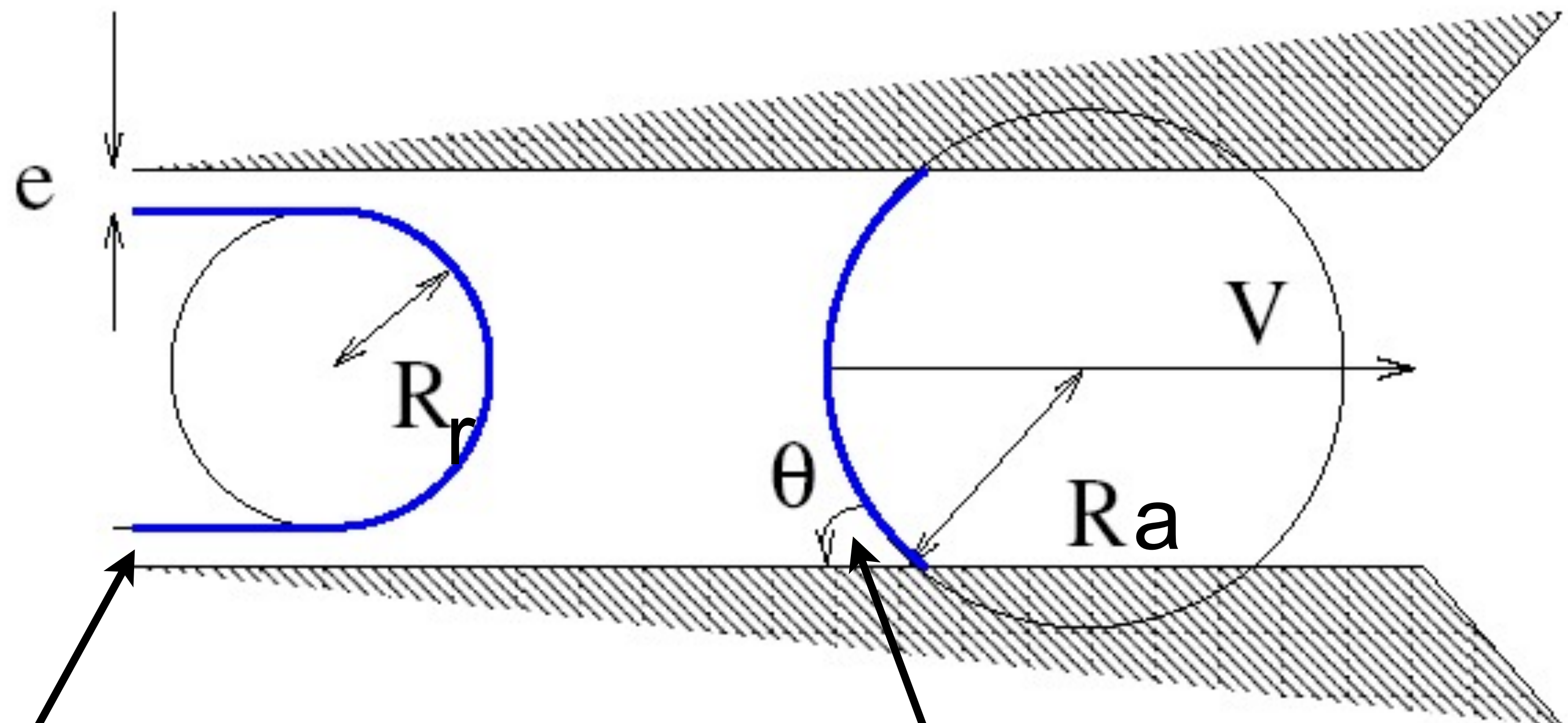




Interfaces are not symmetric: Added pressures

36

# Resistance due to caps

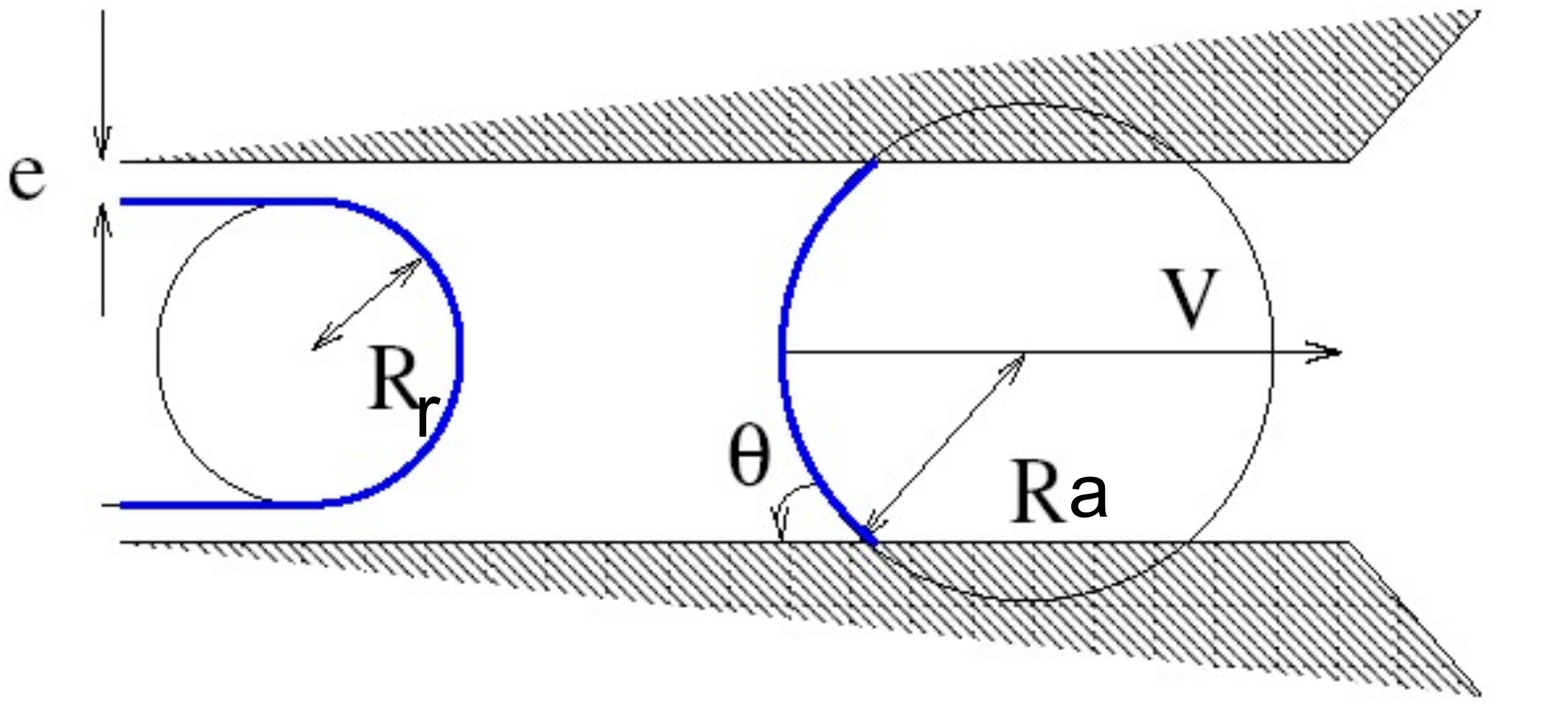
Front and rear caps do not have same curvature



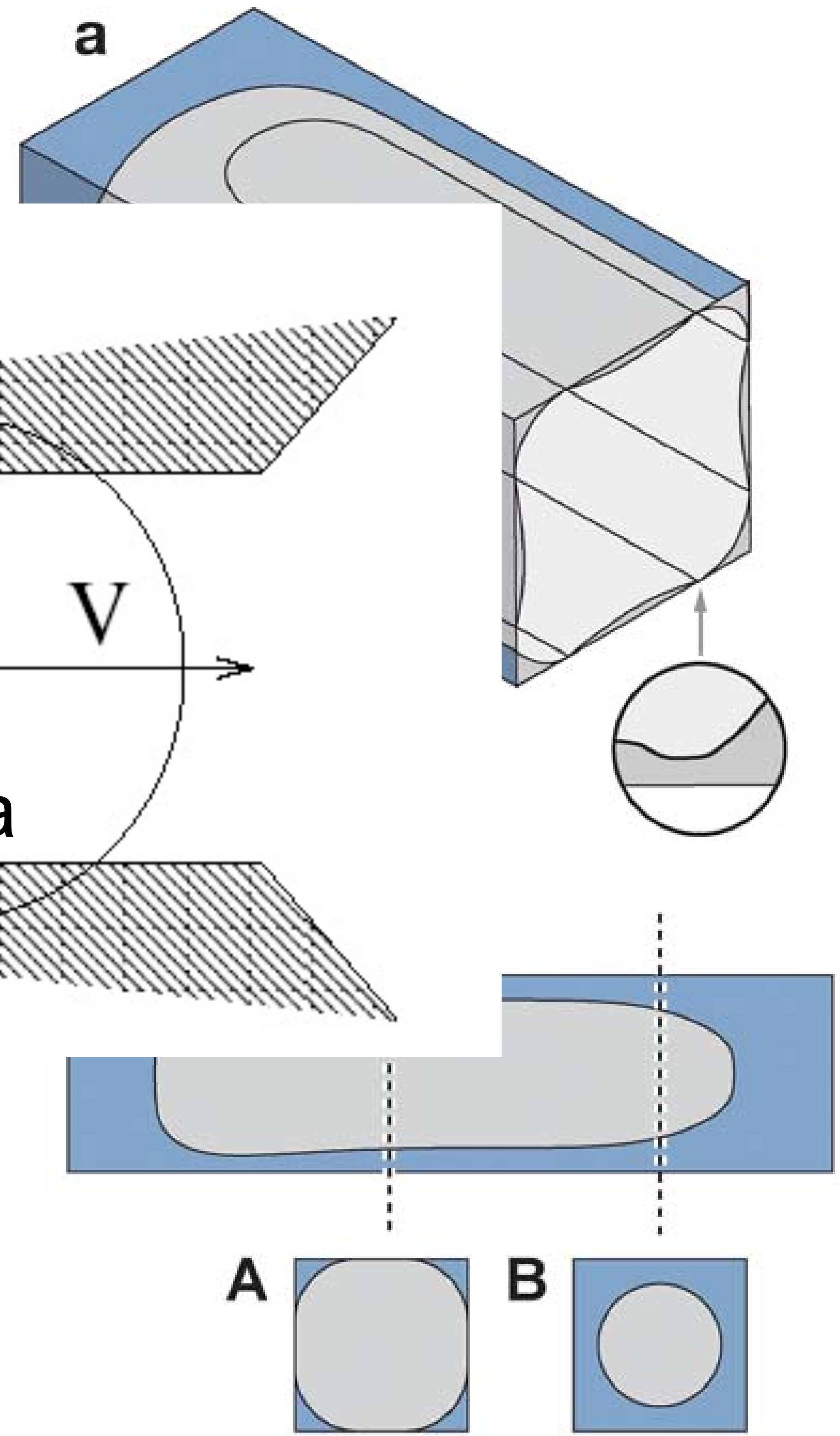

Bretherton film

Dynamic «advancing» contact angle

# Complementary descriptions




Ody et al, 2007

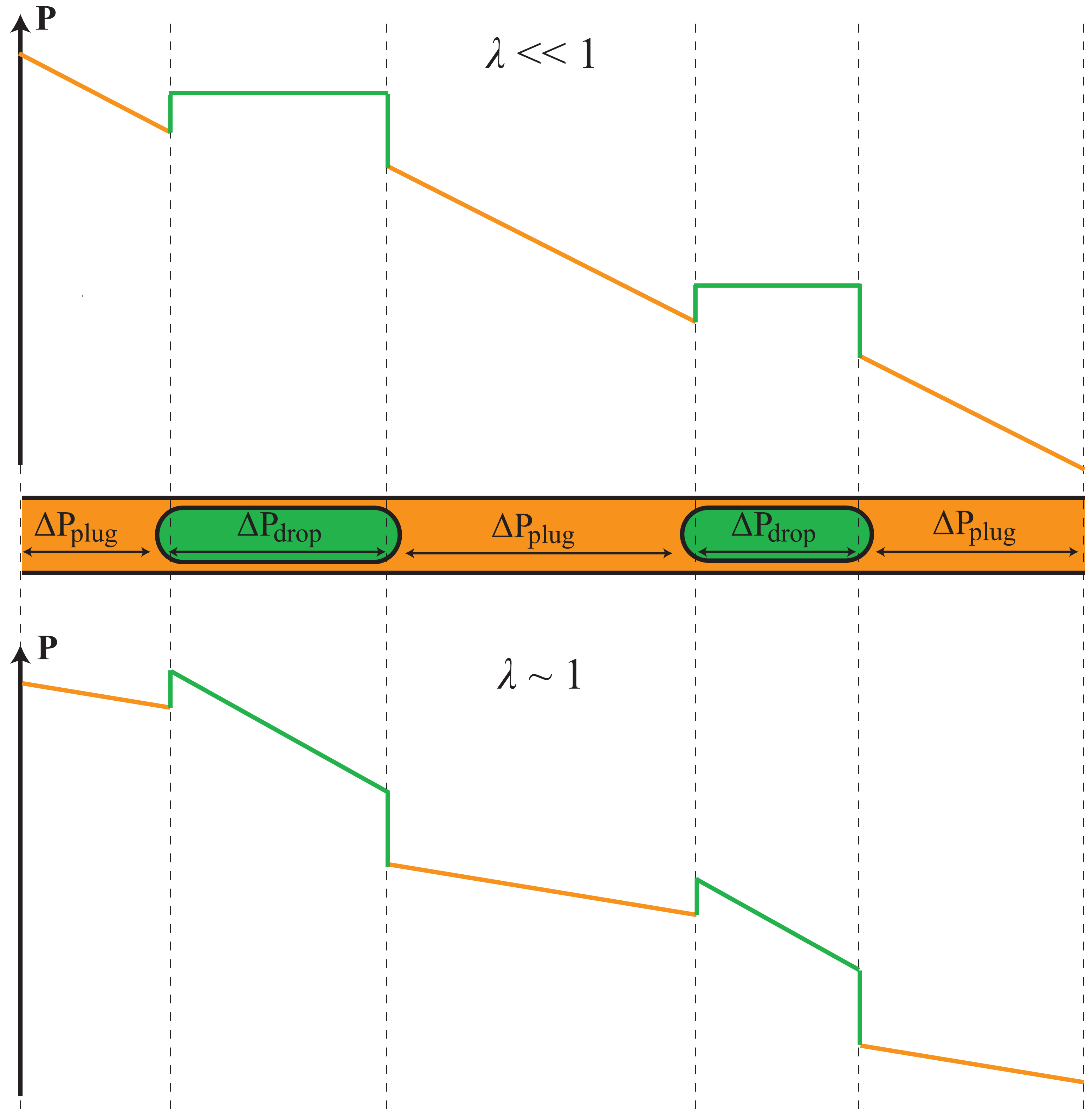



Ajaev & Homsy, 2005

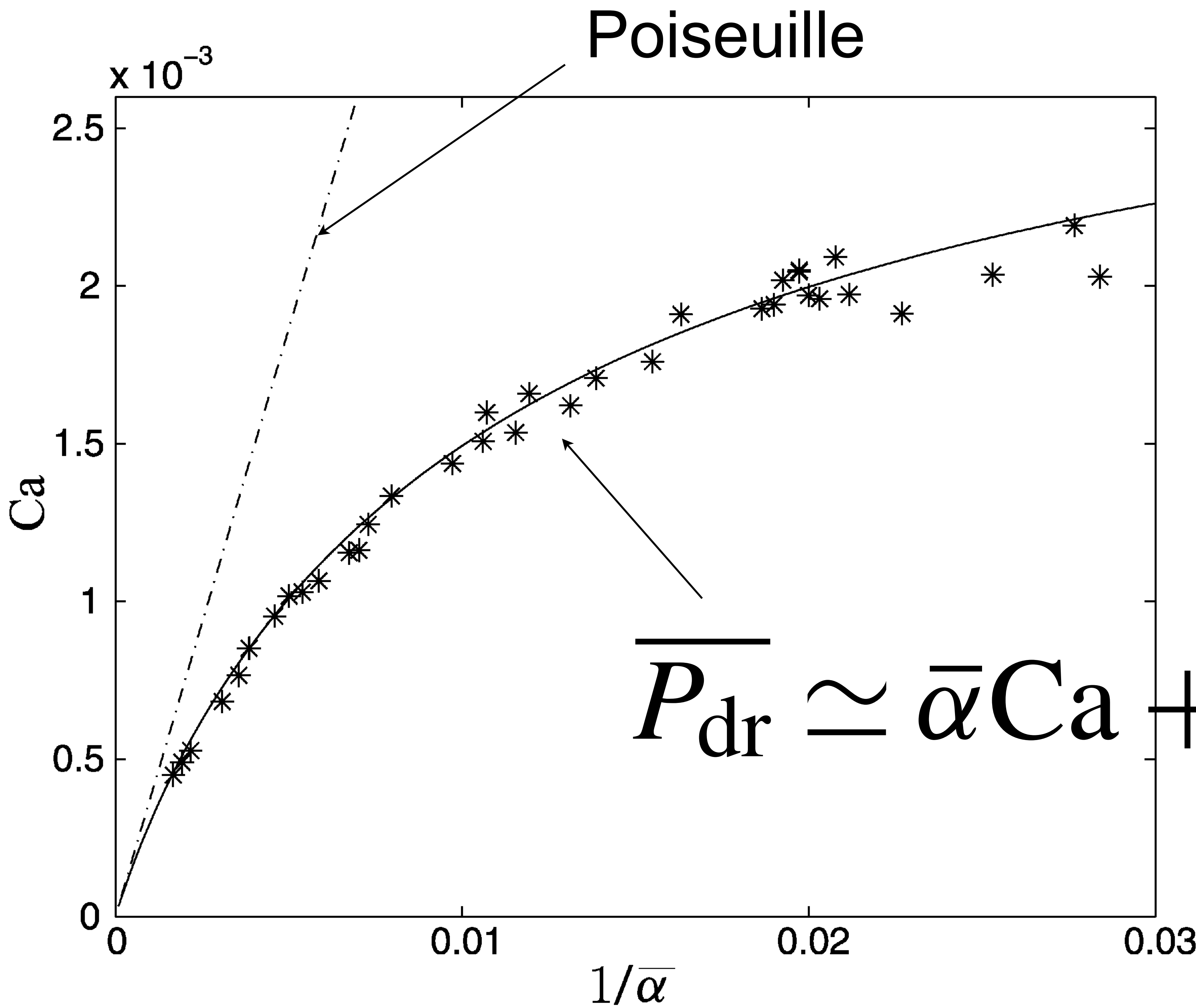
# Combine all resistances

Bubbles

Drops



# Verify pressure-flow rate relation

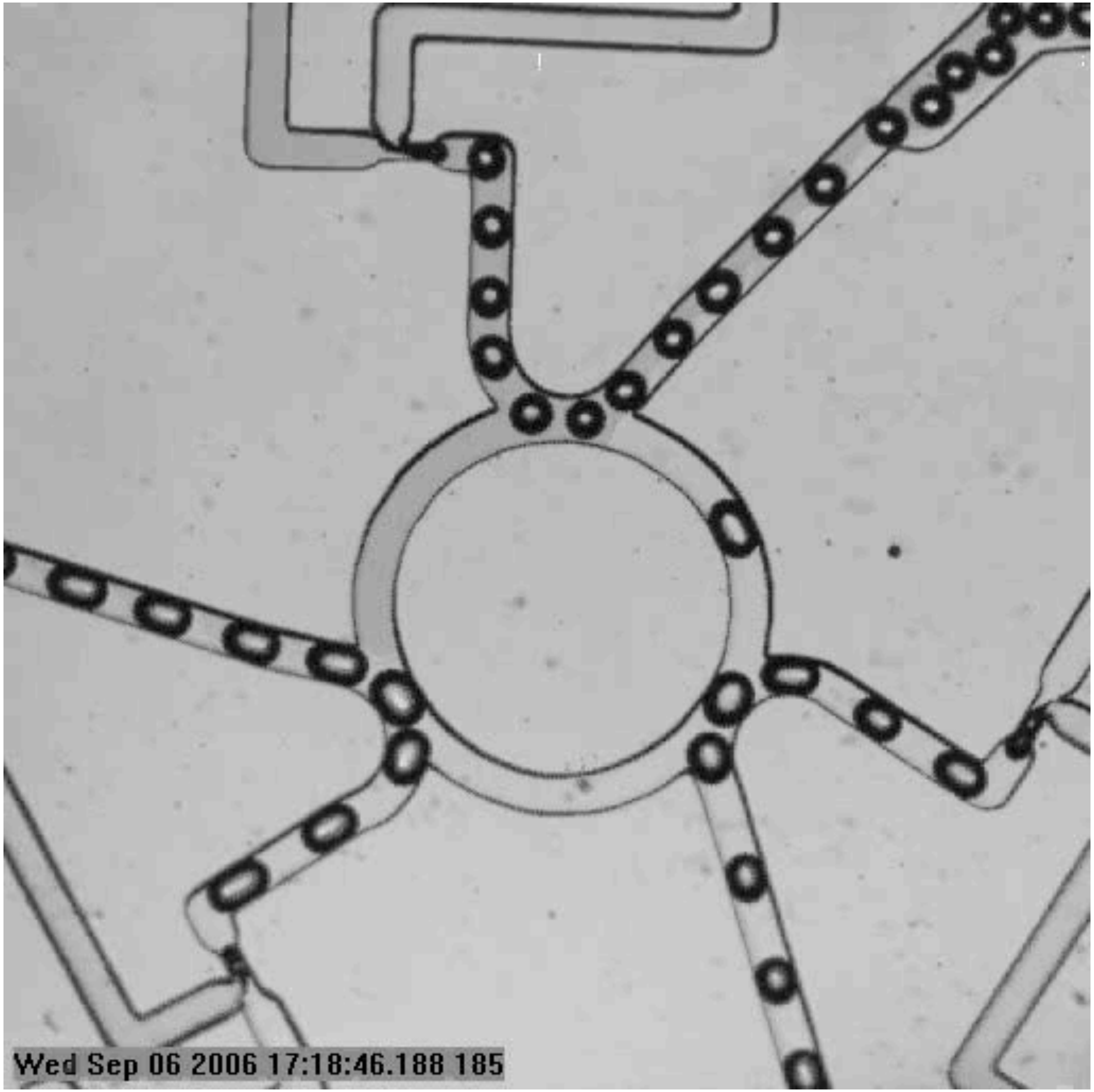



$$\overline{P_{\mathrm{dr}}} \simeq \bar{\alpha}\mathrm{Ca} + \beta \mathrm{Ca}^{2/3}$$

$\bar{\alpha}$~L

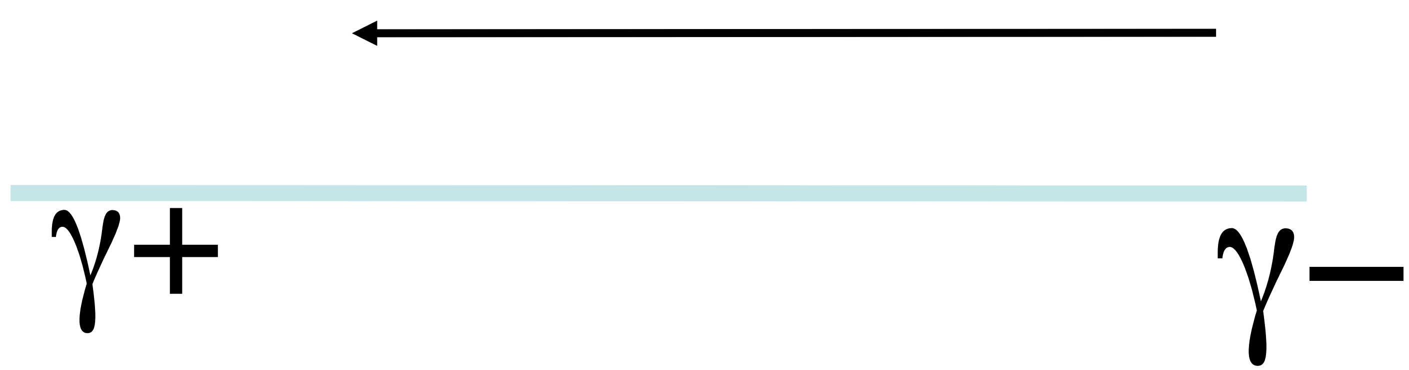
Ody, Baroud & de Langre, 2007

# Added resistance of bubble...



# Surface tension-driven flows

An imbalance of surface tension leads to a flow

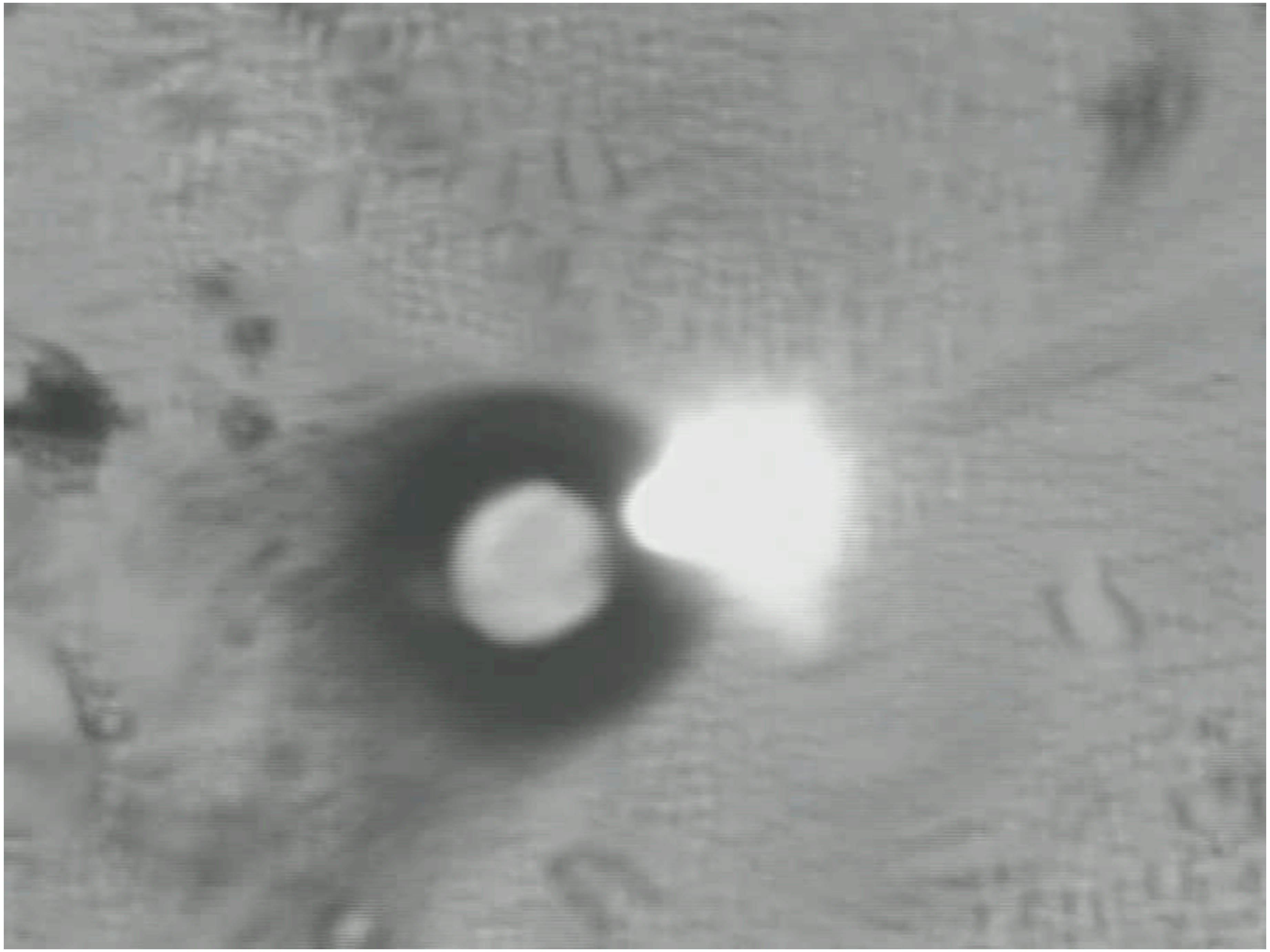


# Surface tension-driven flows

$\gamma$ is a function of temperature

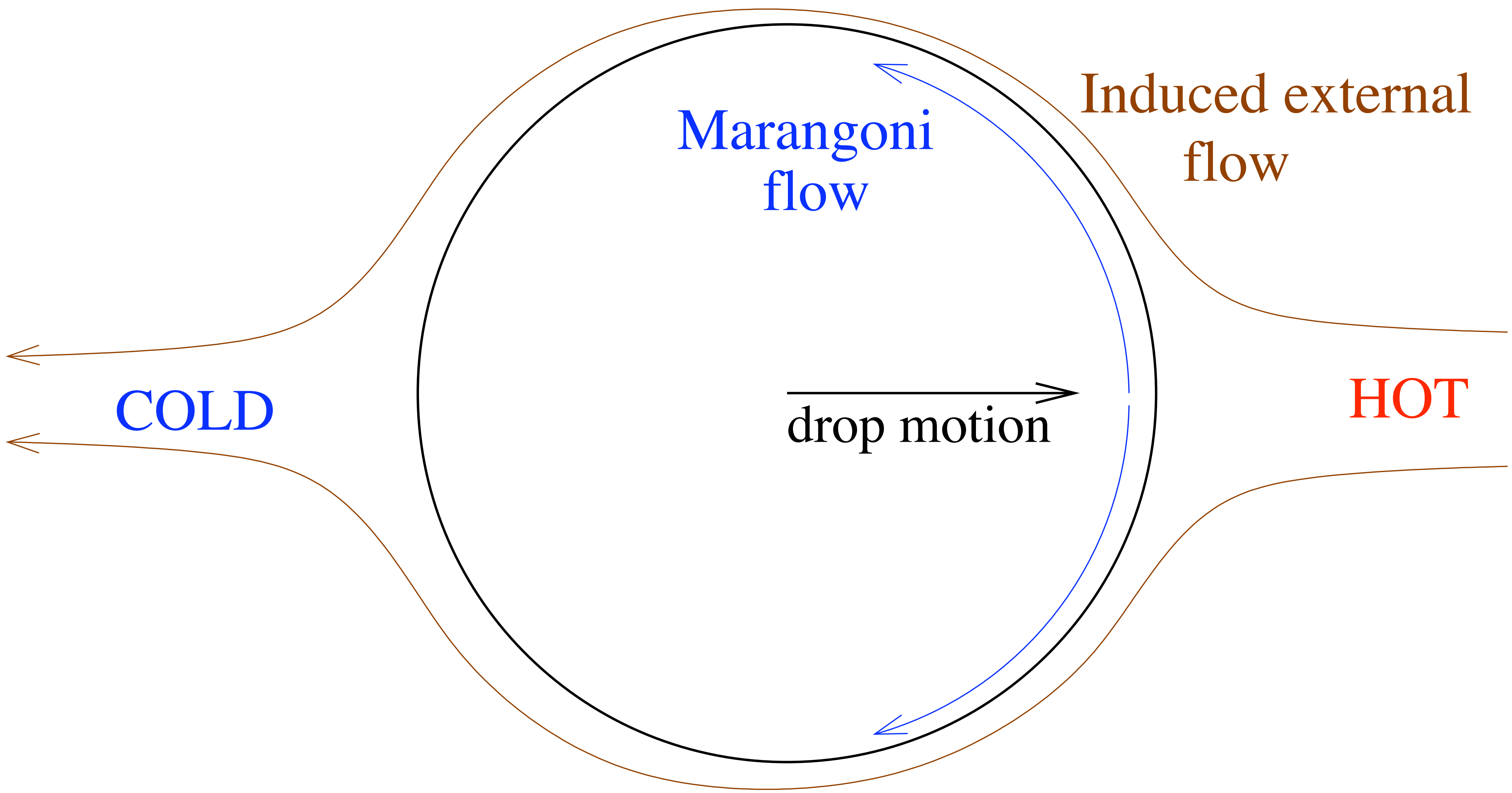
Put an ice cube in oil, with particles on the surface to show the flow

# Thermocapillary flow

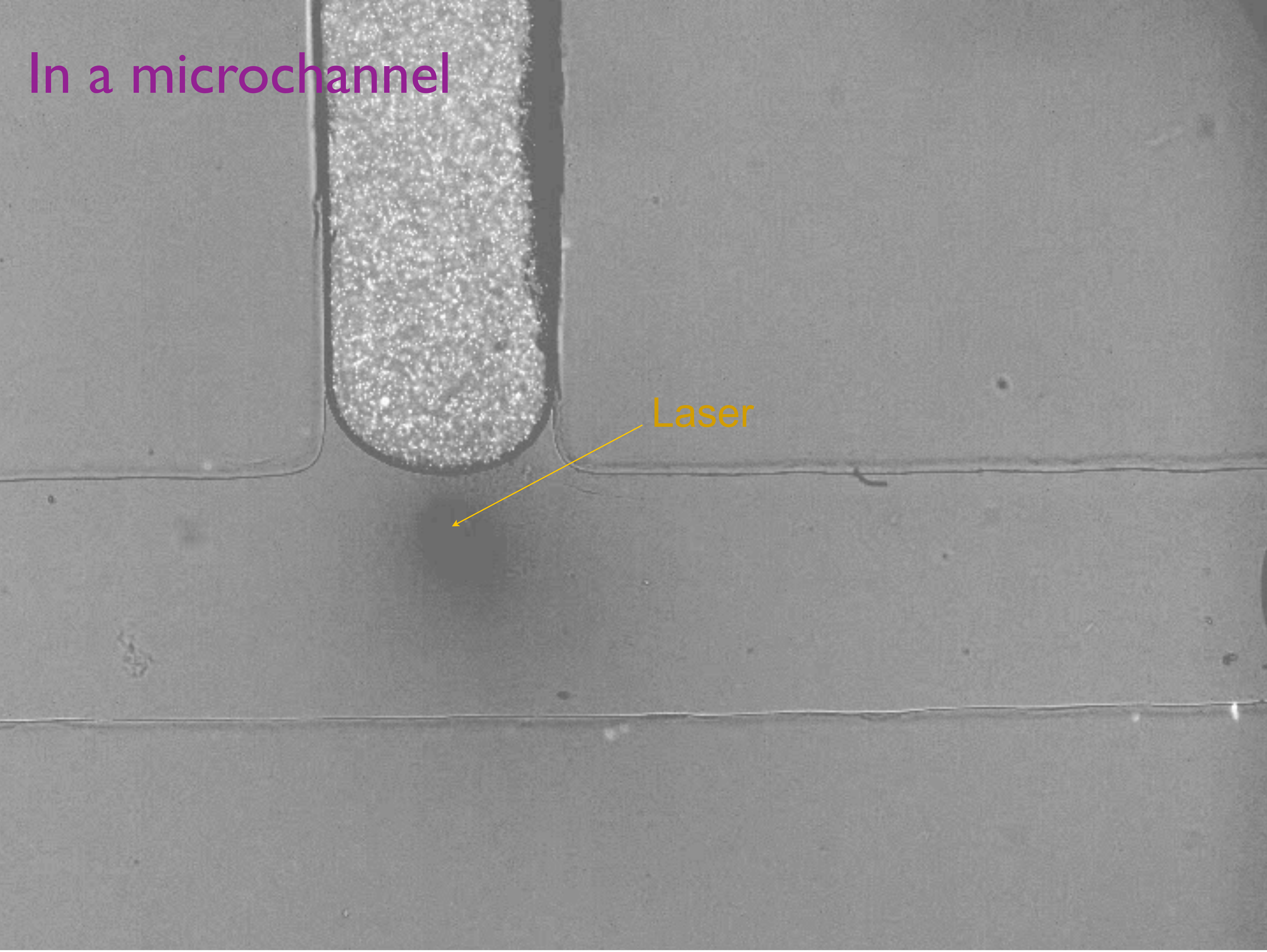


Laser heates a micro-bubble

# Swimming drop

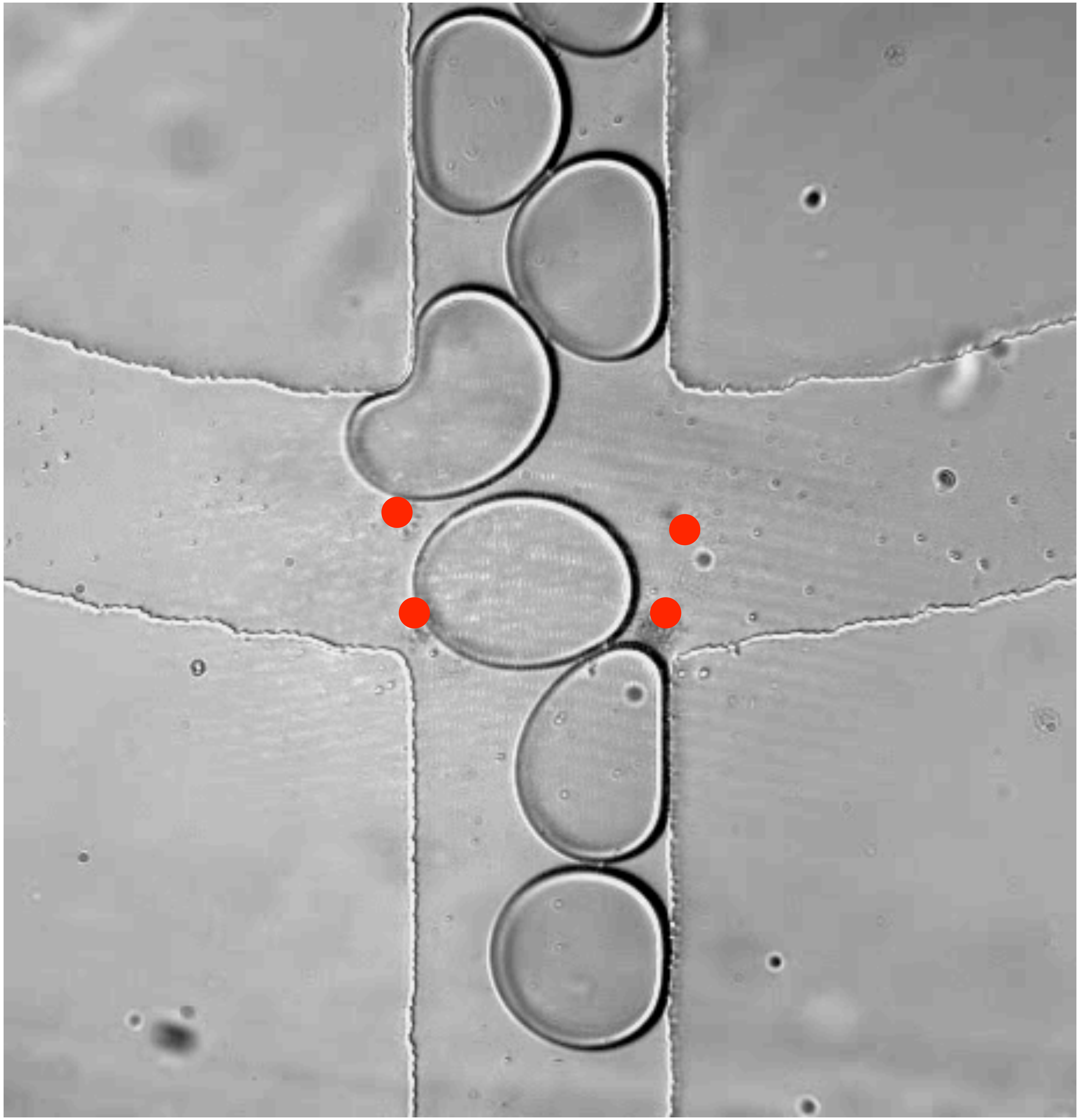


# In a microchannel



# Guiding drops

Wall-less microchannels



# Advanced drop operations

Individual control over each droplet