# Micro-scale flows II- Multiphase flows

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#### The bad news:

### Presence of drops and bubbles introduces nonlinearities into Stokes equation

# Sign of nonlinearity



Oil + surfactant

Cordero & Baroud, unpublished

# Slightly different conditions



Cordero & Baroud, unpublished

#### Surface tension **Surface of drops of the otherwise sequences** Presence of drops or bubbles introduces nonlinearities into the otherwise linear Stokes equa-

#### •Two ways to think about surface tension:  $T$  ingredient that drops provide is surface tension  $\mathcal{L}$  , surface tension  $\mathcal{L}$ the state of intervals in the intervals of  $\sim$ thought of in two complete states of in two complete states  $\frac{1}{\sqrt{2}}$



**•• It's a force per unit length** ‣ Energy per unit area  $\gamma = [N/m]$ **•** Little  $\frac{1}{2}$  per unit area  $\begin{bmatrix} 0 & 1 \end{bmatrix}$ ις per unic lengur **•• It's a set of the drop tends to minimize the drop tends**  $\gamma = \left[\mathrm{J/m}\right]^2$ 

#### γ (1966)<br>1961 - Αντικά Αγγλίας της Αγγλίας<br>1961 - Αντικά Αγγλίας της Αγγλίας ∆ρ*gl*<sup>2</sup> Gives cohesion to liquids

#### **Consequence**  $\epsilon$  consequence tension is that the pressure inside a drop or a bubble inside a drop or a bubble inside a drop or a bubble is higher than  $\epsilon$ than the pressure outside  $\blacksquare$



Pressure inside a bubble larger than pressure outside Pressure in a «plug» lower than outside ρ*U*2*l* larger than pressure outside *Bo* = *n*er tl <sup>γ</sup> (49)

Liquid cylinder: Pressure approach Liquid cylinder: Pressure approach



# **Aggle 12** *What happens if the diameter is*



## A flow takes place from P+ to P-

#### than the pressure outside. Surface tension in the system in the syst Rayleigh-Plateau instability



# |/ Which increases imbalance:

#### Cylindrical jet is always unstable to long wavelengths. One can ask two questions by scaling arguments : Which will appear and how long will be the set of the break  $\mathbb{R}^n$ Cylindrical jet is always unstable to long wavelengths. One can ask two questions by scaling Energy approach



 $V_{jet} = \pi h^2 L$ Jet volume

**Surface area**  $A_{jet} = 2\pi hL$ 

#### and its surface area is : Energy approach









Drop volume  $V_{drops} = N \cdot$ 4 3  $\pi R^3$ 

Surface area  $A_{drops} = N \cdot 4\pi R^2$ 

# Minimize energy



 $Conserve volume,$ compare areas  $R^3 =$ 3*L*  $\frac{\partial L}{\partial N}h^2$  $R^3$   $-$ 3*L*  $\frac{3L}{4N}h^2$ 

$$
\frac{A_{jet}}{A_{drops}} = \frac{2R}{3h}
$$

 $R^* > 3h/2.$ 

Surface area is reduced if:









# Classic problem

• Plateau (1850) presented • Rayleigh (18 this argument and predicted «optimal wavelength» to be Wavelengths that are unstable are therefore λ ∝ *R*<sup>∗</sup> aver *µU*

$$
\lambda = 2\pi h.
$$

- 3*h* So the south is under the the the drops is unstable intervals in the intervals in the intervals in the *R*<br>Array and the articles in the second intervals in the move of the second is a second in the *R*∗ *R*∗ *X* 3*h*/2. • Rayleigh (1879) realized that dynamics must play a role in wavelength selection of  $\lambda > \lambda$ cr igth) to be wavelength s
	- He found λ=9*<sup>h</sup>* **LIE IOUIIU** N-



#### Capillary length Capillary length is what gives considerable control to liquid series and doesn't exist in the case of  $\mathbb{R}^n$ Capillary length



#### Hydrostatic pressure:

**h** (48) Lau et al, 2003 Lau et al, 2003

*Pcap* ∼  $\frac{1}{2}$  $P_{cap} \sim \frac{\gamma}{h}$  $\gamma$  $\frac{1}{h}$ Capillary pressure:  $\mathbb{R}^w$  the two two, we get the capital intervals in the capital int

$$
P_{hs} \sim \rho gh
$$
  
Capillary pressure:  

$$
P_{hs} = P_{cap} \Rightarrow L_C \sim \sqrt{\frac{\gamma}{\rho g}}
$$

### In micro-systems

Size always smaller than *LC*

→ Gravity can always be ignored ∆ρ*gl*<sup>2</sup> r and the set of the s<br>The set of the set of t

Inertial effects can also be ignored

Viscous vs. Capillary competition

$$
Ca = \frac{\mu U}{\gamma}
$$

by considering the viscous stress. For a given fluid pair, *Ca* is a measure of the velocity of flow only

#### Multiphase flows in micro-channels face at every point. For clean and isothermal interfaces, one recovers eqn (1). The relation between g and the local surfactant off. The size of the droplet is set by a competition between the After droplets are produced, they are transported along microiuitiphase flows in micro-char



#### Production<br>
and the interface and eventually leads to drop and eventually leads to drop and eventually leads to drop and e<br>
drop/bubble pinch and eventually leads to drop and eventually leads to drop and eventually leads t face at  $\sim$ recovers eqn (1). The relation between g and the local surfactant  $\blacksquare$



### Drop production





#### Cordero & Baroud, 2010

#### a microchannel, where it encounters the immiscible carrier fluid which is driven in driven in dependent in the independent of the two fluids of the two fluids of the two fluid In microchannels

#### $\epsilon$  are dependently become production in a T-junction in a T-junction in a T-junction. The dispersed production in a T-junction in a T-junction in a T-junction in a T-junction in a T-junction. The dispersed production in phase and the carrier phase meet at 90 degrees in a T-shaped junction. production. In the geometry of Bad news 2: Formation dynamics essentially tension, viscosities) determined by f deforms the interface and eventually leads to drop/bubble pinch determined by the geometry

#### off. The size of the size of the droplet is set by a competition between the droplet is set by a competition b Three classic designs are now standard



Fig. 1 Example of dropped of dropped production in a co-axial injection in a co-axial injection of dropped of dropped of dropped of the state of





#### Multiphase flows in micro-channels After droplets are produced, they are transported along microiuitiphase flows in micro-char flows have received much interest and can be separated into two  $\mathcal{L}$  . Lubrication films and droplet velocity and droplet velocity  $\mathcal{L}$



#### $\mathbf{B}$ How do big drops flow in a microchannel?

### Drop creates recirculation

 $\lambda \ll 1$ 







#### This journal is ª The Royal Society of Chemistry 2010 Lab Chip, 2010, 10, 2032–2045 | 2039  $\mathsf{L}$  Computations in the reference  $\mathsf{L}$ 2D computations Sarrazin et al, 2006

 $(b)$ 

# Recirculating flows



#### Jensen lab, MIT, 2005

#### $\begin{array}{ccc} \hline \end{array}$  between the two halves of the plug. In winding channels of the plug. In winding channels winding chan Can we use this to halve hulds! Can we use this to mix fluids?



Song et al, 2003  $\ldots$ 

# Chaotic mixing





#### Chaotic miving (modern) Chaotic mixing (modern)



Song et al, 2003



# In a plug







#### Jensen lab, MIT, 2005 µm, depth 280 µm). Instantaneous velocity vector and streamline plots are obtained from µPIV measurements, and concentration

#### $t$  $\sim$  carrier fluid  $\sim$  can only be accounted for by the difference How fast do drops flow?



#### Backward flow in film pushes drop faster than mean velocity Rockward flow in film puches drop foster than moan backward now in mini publics drop laster than mean

$$
\frac{V_d - V_{ext}}{V_d} \propto Ca_d^{2/3}
$$

Fig. 4 A Dispersed flow: small droplets immersed in a carrier fluid. B (Drop goes faster than mean velocity!)

See Fairbrother & Stubbs, 1935

## In a rectangular micro-channel



FIGURE 5. Cross-sections of the flow domain for two capillary numbers ( $Ca = 0.02, 1.0$ ), three aspect ratios ( $\alpha = 1, 3, 7$ ) and  $Bo = 0$ , at a distance of  $3.92\alpha$  behind the tip.

deLozar et al. 2008

#### ps leave a fil Drops leave a film behind them



« Bretherton » problem

$$
\frac{e}{H} \propto Ca_d^{2/3}.
$$

Film thickness increases with increasing Capillary number bubble trains in the trains of the train

(Same physics as Landau-Levich film)

#### Cubaud & Ho, 2004

# How fast the drops flow 2

![](_page_30_Figure_1.jpeg)

 $\mathbf{z}$  determines the thickness equation  $\mathbf{z}$  $\frac{1}{\sqrt{2}}$ Rectangular channels: Gutters change everything Slug flow: a succession of plugs and droplets. C Cross-section view of Rectanguiar channels: Gutters change everything

number Cad. a latter modise that derived in a circular capital capital capital capital capital the  $\frac{1}{2}$  $\zeta$ ucccion dicecturation  $\zeta$  view of a large  $\zeta$ slower than mean velocity Flux in gutters directed along Vd, so drops go  $\mathbf{S}^{\text{S}}$  and a order in Cad, which is the effect of films by an order in  $\mathbf{S}^{\text{S}}$  and which is the effect of films by an order in Cad, which is the effect of films by an order in  $\mathbf{S}^{\text{S}}$  and which is th FIUX in gutters directed along vd, so drops go

$$
\frac{V_d - V_{ext}}{V_d} \propto - C a_d^{-1/3}
$$

**Similar scaling results have been derived for moving for moving for moving for moving for moving for moving fo** see Wong, Radke, Morris, 1995<br>N Cross See Wong, Radke, Morris, 1995 See Wong, Radke, Morris, 1995

#### $\mathbf{V}$  value What about in experiments?

![](_page_31_Figure_1.jpeg)

as the liquid phase, we found that, independent of Vnb and Vb,  $\frac{1}{200}$  at all  $\frac{2007}{100}$ system versus the speed of the liquid. In a system in which no Fuerstman et al, 2007

## Detailed measurments

![](_page_32_Figure_1.jpeg)

# Pressure profile in a 2 phase fluid flow

### Divide flow into three parts:

- External fluid flow
- Internal fluid flow
- Interfaces

![](_page_33_Figure_5.jpeg)

# Flow away from interfaces

Assuming distance between plugs is large, use single phase relation: λ ! 2π*r* (55)

$$
\Delta P = \sum_N \mathcal{R}_N Q
$$

For each plug and each drop

![](_page_34_Picture_4.jpeg)

#### of interfaces exactly how thick the depth of correlation of correlations of correlations of correlations of co Effect of interfaces

![](_page_35_Figure_1.jpeg)

medical grade polyethylene tubing (Intramedic, outer diameter Interfaces are not symmetric: Added pressures coated with OTS using a similar procedure as above (5 minutes as above (5 minutes as above (5 minutes as above<br>The coated with OTS using a similar procedure as above (5 minutes as above (5 minutes as above) (5 minutes as Interfaces are not symmetric: Added pressures

<sup>−</sup><sup>1</sup> solution of OTS). The coated sheets

# Resistance due to caps

Front and rear caps do not have same curvature

![](_page_36_Figure_2.jpeg)

# Complementary descriptions

![](_page_37_Figure_1.jpeg)

dimanche 3 avril 2011 38

# Combine all resistances

![](_page_38_Figure_1.jpeg)

#### $\mathbf{b}$ verify pressure-flow obtained from Bretherton's and Tanner's laws. Verify pressure-flow rate relation

![](_page_39_Figure_1.jpeg)

### Added resistance of bubble...

![](_page_40_Picture_1.jpeg)

# Surface tension-driven flows

An imbalance of surface tension leads to a flow

![](_page_41_Figure_2.jpeg)

# Surface tension-driven flows

#### $\gamma$  is a function of temperature

Put an ice cube in oil, with particles on the surface to show the flow

![](_page_42_Picture_3.jpeg)

# Thermocapillary flow

![](_page_43_Picture_1.jpeg)

Laser heates a micro-bubble

# Swimming drop

![](_page_44_Figure_1.jpeg)

# In a microchannel

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### Guiding drops

#### Wall-less microchannels

![](_page_46_Picture_2.jpeg)

### Advanced drop operations

#### Individual control over each droplet

![](_page_47_Picture_2.jpeg)