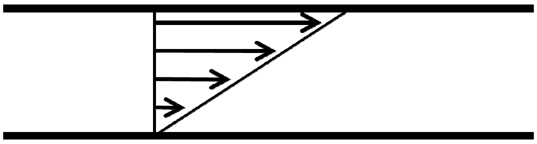
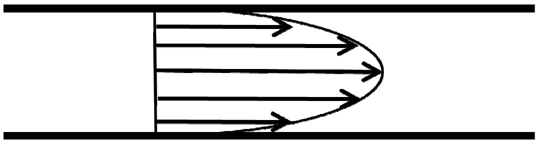
Exercise 1 – Two flow fields

We assume two parallel plates which extend in the (x,y)-plane and have a distance d in z-direction. In between there is a Newtonian fluid. Only linear effects are considered. We assume two flow fields and look for the shear rate as a function of the characteristic velocity.

a) The top plate moves with a speed v_{top}. What is the speed of the fluid as a function of the z-coordinate? What is the shear rate $\partial v/\partial z$?
b) Both plates rest. The speed profile of the fluid is assumed to be parabolic, according to $v(z) = z\cdot(d-z)\cdot v_{max}\cdot 4/d^2$. What is the average speed v_{av} of the fluid? What is the shear rate at the plate surfaces as a function of the average speed v_{av}?





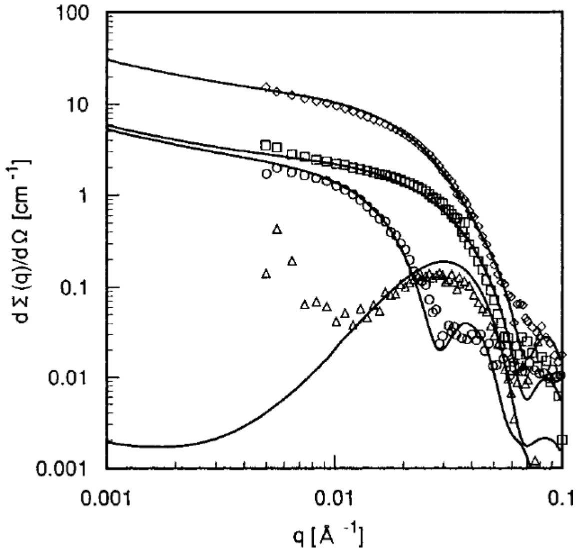
Exercise 2 – Fourier-transformation of a lamellar structure

We assume a symmetric lamellar structure which shall be Fourier-transformed in discrete space. We use the function $f(x) = \begin{cases} +1 \text{ for } 0 \le x < \pi \\ -1 \text{ for } \pi \le x \le 2\pi \end{cases}$. The Fourier Integrals are $F_n = \int_0^{2\pi} f(x)\sin(nx)dx$.
The intensities are proportional to the amplitude squared, i.e. $I_n=F_n^2$. Calculate I_n and F_n! How would the Fourier-transformation be interpreted for a continuous variable q instead of n? How does the 3-dimensional Fourier-transformed lamellar structure look like?

Exercise 3 – reciprocal lattice of a hexagonal structure

We define three vectors to describe the structure of a hexagonal cylinder structure, which read
$\vec{r}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \vec{r}_2 = \begin{pmatrix} 1/2 \\ \sqrt{3/4} \\ 0 \end{pmatrix}, \vec{r}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. In the (x,y)-plane the structure is hexagonal according to the two vectors $\vec{r}_1, \vec{r}_2$. In the z-direction the cylinders should be infinitely long.
Calculate the vectors of the reciprocal lattice, according to $\vec{g}_1 = \vec{r}_2 \times \vec{r}_3$ (and rotating indices). For simplicity we leave the vectors of the reciprocal lattice not normalized. Visualize the real space and reciprocal space picture! What happens for the z-direction in reciprocal space according to the infinitely long cylinders?

Exercise 4 – Scattering of Micelles (Contrast Variation)



The three symbols □, ○, and △ indicate the characteristic small angle scattering of spherical polymer micelles under different important contrast conditions. There are three conditions called: shell contrast, core contrast and zero average contrast. What do these terms specify? Which condition can be connected to which symbol (or curve)? Why?

Exercise 5 – Helfrich energies

The simplest structures for microemulsions are spherical, cylindrical droplets and the lamellar phase. Rationalize the principal curvatures c_1 and c_2 for these structures – they are the extreme curvatures for the considered geometry. The Helfrich free energy reads (assuming the equilibrium curvature c_0 being zero):

$$F = \int dS \cdot \left(\tfrac{1}{2} \kappa (c_1 + c_2)^2 + \bar{\kappa} \cdot c_1 c_2 \right)$$

Since for the considered geometries, the principal curvatures are constant, the normalized free energy is even simpler, and reads:

$$f = \tfrac{1}{2} \kappa (c_1 + c_2)^2 + \bar{\kappa} \cdot c_1 c_2$$

Calculate *f* for the three cases!